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THE STRUCTURE OF SINGULARITIES IN Φ-MINIMIZING NETWORKS IN $\mathbf{R}^2$

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It is well known that length-minimizing networks in $\mathbf{R}^2$ consist of segments meeting only in threes. This paper considers uniformly convex norms Φ more general than length. The first theorem says that for any such smooth Φ, minimizing networks still meet only in threes. The second theorem shows that for some *piecewise* smooth Φ, segments can meet in fours (although never in fives or more).

1. Introduction. Length-minimizing networks in $\mathbf{R}^2$ consist of straight line segments meeting only in threes. Soap films meet in threes for exactly the same reasons. (See Figures 1.0.1 and 1.0.2 and [**CR**, pp. 354–356].)

This paper studies the structure of minimizing networks for elliptic integrands Φ, which depend on direction and thus are more general than length. (The surface energy of most crystals, unlike that of soap films, depends on orientation as well as area.)

THEOREM (3.3). *Let Φ be a smooth, elliptic integrand. Then, segments in Φ-minimizing networks meet only in threes.*

THEOREM (3.4). *There is a piecewise smooth, elliptic integrand Φ_0 for which the X is Φ_0-minimizing* (*see Figure* 1.0.3).

Theorem 3.3 is proved by showing that conjunctions of more than three segments are unstable. The proof of Theorem 3.4 uses symmetry arguments to reduce the analysis to a one-dimensional calculus problem. The result holds for an infinite family of elliptic integrands with unit balls that are perturbations of the square. (The unit ball is the set of all points reachable from the origin with an energy no greater than one.) (See Figure 1.0.4.) The square itself is the unit ball of the rotated "Manhattan Metric," Φ_M, for which our result would be trivial; however, Φ_M is not elliptic because the square is not *uniformly* convex (see §2).