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EQUIVARIANT NIELSEN FIXED POINT THEORY FOR G-MAPS

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Let $f: V \to X$ be a G-map defined on an open invariant subset V of a G-ENR X where G is a compact Lie group and the G-action on V is not necessarily free. In this paper, we introduce the notion of an equivariant Nielsen number $N_G^c(f, V)$ which is an ordered k-tuple that depends on the isotropy types $(H_1), \ldots, (H_k)$ of V. When G is finite, $N_G^c(f, V)$ gives a lower bound for the minimal number of fixed points in the (restricted) G-homotopy class of f and this lower bound is sharp when the G-action on V is free. We relate $N_G^c(f, V)$ to a local equivariant obstruction to G-deforming a map to be fixed point free and we discuss the relationship between the equivariant Nielsen number and the ordinary Nielsen number.

1. Preliminaries. Let G be a topological group and X be a (left) G-space. For any subgroup H of G, we denote by NH the normalizer of H in G and by $WH = NH/H$, the Weyl group of H in G. The conjugacy class of H denoted by (H) is called the orbit type of H. If $x \in X$, then G_x denotes the isotropy subgroup of x, i.e., $G_x = \{g \in G | gx = x\}$. For each subgroup H of G, $X^H = \{x \in X | hx = x \text{ for all } h \in H\}$ and $X_H = \{x \in X | G_x = H\}$. An orbit type (H) is called an isotropy type of X if H appears as an isotropy subgroup of some x in X. Suppose X has a finite set of isotropy types denoted by $\{(H_i)\}$. If (H_j) is subconjugate to (H_i), we write $(H_j) \le (H_i)$. We can choose an *admissible ordering* on $\{(H_j)\}$ so that $(H_j) \le (H_i)$ implies $i \le j$. Then we have a filtration of G-subspaces $X_1 \subset \cdots \subset X_k = X$ where $X_i = \{x \in X | (G_x) = (H_j) \text{ for some } j \le i\}$. Also, $X_{(H)} = GX_H = X_i - X_{i-1}$ with $(H) = (H_i)$. By a free G-subset of X, we mean a G-invariant subset on which the action is free.

Let G be a compact Lie group. A G-space X is a *G-absolute neighborhood retract* (G-ANR), if X is a metric space and for any G-embedding $h: X \to Y$ is a metric G-space Y such that $h(X)$ is closed in Y, the image $h(X)$ is a G-retract of some open invariant neighborhood in Y. If X is a G-ANR then X^H is an ANR for every closed subgroup $H \le G$. Moreover, if Y is a G-ANR and $f: X \to Y$ is a G-equivalence then $f^H = f|X^H: X^H \to Y^H$ is a