PACIFIC JOURNAL OF MATHEMATICS
Vol. 150, No. 1, 1991

DIHEDRAL GROUP ACTIONS ON HOMOTOPY COMPLEX PROJECTIVE THREE SPACES

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Let D_{2m} be the dihedral group of order $2m$. Given an odd prime m such that the projective class group of D_{2m} has 2-rank $= 0$, we construct smooth D_{2m}-actions on an infinite number of pairwise non-diffeomorphic (distinguished by Pontryagin class) manifolds each of which is homotopy equivalent to CP^3. This is accomplished by applying equivariant surgery theory to normal maps created by an equivariant transversality argument.

1. Introduction. The question which we deal with here is: "Which finite groups can act on differentiably non-standard homotopy $\mathbf{C}P^3$'s?" We use equivariant surgery theory to construct dihedral group actions on an infinite number of differentiably distinct smooth manifolds which are homotopy equivalent to $\mathbf{C}P^3$.

According to [MY], there is a one-to-one correspondence between the integers and the set (actually, it is a group) of diffeomorphism classes of 6-dimensional, smooth, closed manifolds which are homotopy equivalent to $\mathbf{C}P^3$ (such manifolds shall hereafter be called homotopy $\mathbf{C}P^3$'s). For every integer k, there is a unique homotopy $\mathbf{C}P^3$, denoted X_k, with first Pontryagin class $P_1(X_k) = (4+24k)x^2$, where $x \in H^2(X_k)$ is a generator. Then, X_0 is the standard $\mathbf{C}P^3$. All actions shall be effective and smooth.

Some information is known about smooth finite group actions on homotopy $\mathbf{C}P^3$'s. For instance, infinitely many homotopy $\mathbf{C}P^3$'s admit a $\mathbf{Z}_m$-action for almost every prime number m. (For this, and other interesting results, we refer the reader to [DM].) On the other hand, in [M1], it is shown that if X_k admits a smooth, effective $\mathbf{Z}_m \times \mathbf{Z}_m \times \mathbf{Z}_m$-action, for any odd prime m, then $k = 0$, i.e., $X_k = \mathbf{C}P^3$. (There is a more restricted version of this result for $m = 2$. For information about involutions on homotopy $\mathbf{C}P^3$'s, we refer the reader to [DMS].)

In this paper, we shall consider the case of dihedral group actions. To my knowledge, these are the first examples of non-abelian group