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FAMILIES OF RIEMANN SURFACES OVER THE PUNCTURED DISK

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We study holomorphic families of compact Riemann surfaces over the punctured unit disk. For every genus $p \geq 3$ we define a family whose relative canonical bundle has no roots of order $n > 2$. The monodromy group of that family is generated by a product of powers of commuting Dehn twists. We give necessary and sufficient conditions for such a product to generate the monodromy group of a family over the punctured disk.

1. Introduction. By definition an nth root (or root of order n) of a holomorphic line bundle K over a complex manifold M is a holomorphic line bundle $L \to M$ such that the line bundles $L^{\otimes n} \to M$ and $K \to M$ are equivalent. (Every line bundle in this paper will be holomorphic, and equivalence will always be holomorphic equivalence.) In this paper, which continues the study initiated by the second author in **[15]** and **[17]**, we shall be interested primarily in nth roots of the relative canonical bundle $K_{\text{rel}}(V) \to V$ of a holomorphic family $\pi: V \to B$ of compact Riemann surfaces $p \geq 2$. Readers who are unfamiliar with holomorphic families and their relative canonical bundles can find the basic facts about them in §1 of **[17]**. The most important feature of $K_{\text{rel}}(V) \to V$ is that its restriction to each fiber $X_t = \pi^{-1}(t) \subset V$ is the canonical line bundle of the Riemann surface X_t.

For every genus $p \geq 3$, examples were given in **[17]** of holomorphic families whose relative canonical bundles have no roots of order $n > 2$. The base space of each of these families is a quotient space of the Teichmüller space T_p of compact Riemann surfaces of genus p, and has complex dimension $3p - 3$. The simplest base space for such examples would be the punctured unit disk

$$\Delta' = \{z \in \mathbb{C};\, 0 < |z| < 1\}.$$

This paper presents examples with base space Δ' for every genus $p \geq 3$. We define the required families in the next section.

To prove Theorem 1, which states that the relative canonical bundles of these families have no roots of order $n > 2$, we study their