CROSSED PRODUCTS AND GENERALIZED INNER ACTIONS OF HOPF ALGEBRAS

William Chin

This paper examines crossed products $R * H$ where the Hopf algebra H acts weakly on the algebra R and is twisted by a Hopf cocycle t. Invertible cocycles are discussed and a related sort of weak action which we call "fully invertible" is introduced. This condition allows us to undo the action of H in a useful way and allows reasonable behavior of ideals in crossed products. Many crossed products of interest are of this type, including crossed products of cocommutative Hopf algebras with invertible cocycles, crossed products of irreducible Hopf algebras, and all smash products with bijective antipode. We construct the quotient ring Q of an H-prime ring and discuss actions which become inner when extended to Q. This is then applied to describe prime ideals in crossed products over H-prime rings with extended inner actions and it is shown that some of these crossed products are semiprime.

Introduction. This paper involves crossed products $R * H$ where the Hopf algebra H acts on the algebra R and the image of H is twisted by a Hopf cocycle t. The ideas here build on some of those introduced in [**BCM**] and that paper serves as a foundation for what is done here. Under some fairly general technical hypotheses we examine the behavior of ideals and Martindale quotient rings in relation to the weak action of H. Using the quotient ring and results from [**Ch**] and [**BCM**] as main tools we focus on prime ideals in crossed products and then show that certain crossed products with extended inner actions are semiprime.

In the first section crossed products with invertible cocycles are discussed. We introduce a sort of weak action which we call "fully invertible". This condition allows us to undo the action of H in useful ways and allows for reasonable behavior of ideals in crossed products. Many crossed products of interest are fully invertible, including all crossed products with invertible cocycles and cocommutative Hopf algebras H, and all smash products with bijective antipode. Starting with Proposition 1.2, some basic facts concerning ideals in crossed products with fully invertible actions are established.

We introduce H-prime and H-invariant ideals and prove Lemma 1.7, establishing a useful construction of H-invariant ideals when the