PACIFIC JOURNAL OF MATHEMATICS
Vol. 151, No. 1, 1991

# SURGERY WITH FINITE FUNDAMENTAL GROUP I: THE OBSTRUCTIONS

R. JAMES MILGRAM

**This paper determines the surgery obstructions for all surgery problems of the form**

$$\mathrm{id}\times\sigma\colon M^2\times K^{4k+2}\to M^n\times S^{4k+2}$$

**as explicit elements in the surgery obstruction groups $L^h_{n+2}$ where $\sigma\colon K^{4k+2}\to S^{4k+2}$ is the usual Kervaire problem and $M^n$ is a closed, compact, oriented manifold with $\pi_1(M)$ finite. Due to the well known observation that the surgery obstruction for a surgery problem on a *closed manifold* depends only on the resulting cobordism class in $\Omega_*(B_{\pi_1(M)}\times G/TOP)$, this is the fundamental step in obtaining the surgery obstructions for all surgery problems over closed manifolds, as long as $\pi_1(M)$ is finite. (In the case $\pi_1(M)$ infinite, the situation is much more complex. A key part of the question would be resolved if one could prove the Novikov conjecture though.)**

**One of our main results is that only three types of obstruction can occur. This is, in fact, the first step in proving the oozing conjecture. The proof is completed in part II of this paper where we give characteristic class formulae for evaluating these obstructions.**

In 1965 Dennis Sullivan proved a surgery product formula which made it possible to write down characteristic class formulae for evaluating the surgery obstructions of degree 1 normal maps over *simply connected* manifolds. Then in [**W2**] C. T. C. Wall made a preliminary study of the problem of characteristic class formulae for surgery in case the base is a closed manifold with *finite* fundamental group. He was able to show that formulae similar to the Sullivan result must exist in theory, and, moreover, are completely determined if they are known when the fundamental group is a finite 2-group.

W. Pardon [**P**] and G. Carlsson-R. J. Milgram [**C-M**] showed that if one looked at the surgery obstruction groups $L^p_k(\mathbf{Z}\pi)$ relevant to the problem obtained after crossing a given surgery problem with a circle $S^1$ then these groups are determined entirely by the rational representation ring of $\pi$ whenever $\pi$ is a finite 2-group. Based on this I. Hambleton [**H**], and independently B. Williams and Larry Taylor [**T-W**], gave an explicit identification of all possible obstructions for the $S^1\times(\ )$ problem, and showed that there were examples for which each was realized. However, the general case remained out of reach.