PACIFIC JOURNAL OF MATHEMATICS
Vol. 151, No. 2, 1991

ANY BLASCHKE MANIFOLD OF THE HOMOTOPY TYPE OF $\mathbf{C}P^n$ HAS THE RIGHT VOLUME

C. T. YANG

Dedicated to Professor S. S. Chern

The aim of this paper is to prove the result stated in the title.

By a *Blaschke manifold* [**1**, p. 135], we mean a connected closed Riemannian manifold which has the property that the cut locus of each of its points, when viewed in the tangent space, is a round sphere of a constant radius. It is well known that in any Blaschke manifold, all geodesics are smoothly simply closed and have the same length. The canonical examples of a Blaschke manifold are the unit n-sphere S^n, the real, complex, quaternionic projective n-spaces $\mathbf{R}P^n$, $\mathbf{C}P^n$, $\mathbf{H}P^n$ and the Cayley projective plane $\mathbf{Ca}P^2$ with their standard Riemannian metric. These Blaschke manifolds will be referred to as the *canonical* Blaschke manifolds. For general informations on Blaschke manifolds, see [**1**].

The Blaschke conjecture says that *any Blaschke manifold, up to a constant factor, is isometric to a canonical Blaschke manifold*. This conjecture looks plausible, because it has been shown in [**3**, **7**] that any Blaschke manifold either is diffeomorphic to S^n or $\mathbf{R}P^n$, or is of the homotopy type of $\mathbf{C}P^n$, or is a 1-connected closed manifold having the integral cohomology ring of $\mathbf{H}P^n$ or $\mathbf{Ca}P^2$. However, so far it has been proved only for spheres and real projective spaces [**2**, **6**, **8**, **9**].

One crucial step in the proof of the Blaschke conjecture for spheres is to show that any Blaschke manifold diffeomorphic to S^n has the right volume. Hence we formulate the weak Blaschke conjecture [**10**] which says that *any Blaschke manifold has the right volume.*

Let M be a d-dimensional Blaschke manifold, UM the space of unit tangent vectors of M and CM the space of oriented closed geodesics in M. Then UM and CM are oriented connected smooth manifolds and there is a natural oriented smooth circle bundle $\pi: UM \to CM$. In [**8**], it is shown that, if e denotes the Euler class of this