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ON THE ALGEBRAIC PART OF AN ALTERNATING LINK

MORWEN B. THISTLETHWAITE

A simple method is given for determining the algebraic part of an alternating link. It is proved that the only alternating diagrams of elementary links are the "obvious" ones.

1. Introduction. For each classical link L in the 3-sphere, whose complement is irreducible and geometrically atoroidal, it is explained in [**B-S**] how the pair (S^3, L) admits a decomposition, unique up to isotopy, into an algebraic part $(A, L \cap A)$ and a *non-algebraic part* $(N, L \cap N)$. The algebraic part A is constituted, in a manner explained in §2 below, from so-called "elementary tangles" in (S^3, L); N is just the closure in S^3 of $S^3 - A$. As proved in [**B-S**], if the complement of L is not a Seifert manifold, the submanifold A of S^3 may be characterized in more general terms as follows: (i) ∂A meets L transversely; (ii) if $\pi\colon \widetilde{X} \to S^3$ is the 2-fold covering of S^3 branched along L, we may choose the characteristic variety V of $\widetilde{X}$ so that $\pi^{-1}(A)$ is precisely the union of the closed-up Seifert fibered components of $\widetilde{X} - V$. If the complement of L is a Seifert manifold, A is either empty or equal to S^3 (see [**B-S**] for full details).

Since the characteristic variety V of $\widetilde{X}$ consists of incompressible tori and $S^3 - L$ is atoroidal, each component of $\partial A = \partial N$ is a 2-sphere meeting the link L transversely in four points. The link L is said to be *algebraic*, or *arborescent*, if the algebraic part is the whole of S^3.

The main purpose of this article is to describe and justify a simple method for determining the algebraic part of (S^3, L) in the case where the link L is presented as an alternating link. It is proved in [**M1**] that if L admits a connected, alternating diagram which is prime (in the 2-dimensional sense), then $S^3 - L$ is irreducible and geometrically atoroidal. Therefore, in this paper, we shall be concerned with precisely those links admitting connected, prime, alternating diagrams. The method for finding the algebraic part of (S^3, L) is based largely on Menasco's "visibility" results for Conway spheres [**M1**], and has already been investigated by F. Bonahon and L. Siebenmann.