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SINGULARITY OF THE RADIAL SUBALGEBRA OF $\mathscr{L}(F_N)$ AND THE PUKÁNSZKY INVARIANT

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Let $\mathscr{L}(F_N)$ be the von Neumann algebra of the free group with N generators $x_1, \dots, x_N$, $N \geq 2$ and let A be the abelian von Neumann subalgebra generated by $x_1 + x_1^{-1} + \cdots + x_N + x_N^{-1}$ acting as a left convolutor on $l^2(F_N)$. The radial algebra A appeared in the harmonic analysis of the free group as a maximal abelian subalgebra of $\mathscr{L}(F_N)$, the von Neumann algebra of the free group. The aim of this paper is to prove that A is singular (which means that there are no unitaries u in $\mathscr{L}(F_N)$ excepting those coming from A such that $u^*Au \subseteq A$). This is done by showing that the Pukánszky invariant of A is infinite, where the Pukánszky invariant of A is the type of the commutant of the algebra $\mathscr{A}$ in $B(l^2(F_N))$ generated by A and $x_1 + x_1^{-1} + \cdots + x_N + x_N^{-1}$ regarded also as a right convolutor on $l^2(F_N)$.

1. Introduction. Let M be a type II_1 factor with trace τ, $\tau(1) = 1$ and $A \subseteq M$ a maximal abelian von Neumann subalgebra (briefly a M.A.S.A.). Following J. Dixmier [**1**], let $N_M(A) = \{u \in M \mid u \text{ unitary}, uAu^* = A\}$ be the normalizer of A in M and $B = N_M(A)''$ the von Neumann subalgebra generated by $N_M(A)$ in M. According to the size of B in M, A is called singular if $B = A$ and A is called regular (or Cartan) if $B = M$. While examples of regular M.A.S.A.'s are readily available by the classical group measure space construction from a free action of a discrete group on a measure space, examples of singular M.A.S.A.'s are more difficult to obtain (see, e.g., [**1**], [**6**], [**9**], [**10**], [**5**]).

The aim of this paper is to show that in the von Neumann algebra $M = \mathscr{L}(F_N)$ of the free group with N generators $X_1, X_2, \dots, X_N$, the radial algebra (i.e. the abelian von Neumann subalgebra generated by $X_1 + \cdots + X_N + X_1^{-1} + \cdots + X_N^{-1}$) is singular. This algebra has been studied intensively in [**2**], [**3**], [**7**] because of its connections with the problem of computing spectra of convolutors and with representation theory of F_N. In particular in [**7**] it is shown that the radial algebra is a M.A.S.A. in $\mathscr{L}(F_N)$.

To prove our result we need in fact to prove more than the singularity of A. In order to express this we recall some definitions.