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THE PENNEY-FUJIWARA PLANCHEREL FORMULA FOR ABELIAN SYMMETRIC SPACES AND COMPLETELY SOLVABLE HOMOGENEOUS SPACES

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A distribution theoretic version of the Plancherel formula for the decomposition of the quasi-regular representation of a Lie group G on $L^2(G/H)$ is presented. The formula is proven in two situations wherein the irreducible representations that occur in the decomposition are monomial. The intertwining operator that effects the decomposition is derived in terms of integral operators that arise from the distributions.

1. Introduction. We are concerned here with the quasi-regular representation $\tau = \operatorname{Ind}_H^G 1$ for G a connected Lie group and H a connected closed subgroup. The Orbit Method instructs us as to how to decompose such a representation into irreducibles. Indeed, if there is a "nice" orbital parameterization for the dual $\widehat{G}$, or at least for the part $\widehat{G}_H = \{\pi \in \widehat{G}: \pi$ is weakly contained in $\tau\}$ that supports the quasi-regular representation, then the Orbit Method [11] suggests the direct integral decomposition:

$$\tau = \int_{\mathfrak{h}^\perp/H}^{\oplus} \pi_\varphi \, d\dot{\varphi} = \int_{G\cdot\mathfrak{h}^\perp/G}^{\oplus} n_\varphi \pi_\varphi \, d\tilde{\varphi}, \tag{1.1}$$

where $\mathfrak{h}^\perp = \{\varphi \in \mathfrak{g}^*: \varphi(\mathfrak{h}) = 0\}$, $n_\varphi = \#[G\cdot\varphi \cap \mathfrak{h}^\perp]/H$, and $d\dot{\varphi}$, $d\tilde{\varphi}$ are push-forwards of Lebesgue measure. (Here $\mathfrak{g} = \operatorname{Lie}(G)$ and g^* is the real linear dual.) Such an orbital decomposition is valid if G is simply connected exponential solvable [6], [13], or G/H is abelian symmetric [9], [11], or Riemannian symmetric [11] (see [11] for other cases as well). The direct integral formula (1.1) is "soft" in that it provides the abstract representation theoretic decomposition of τ—i.e., it describes the spectrum, spectral multiplicity and spectral measure. But is is not "hard" in that it avoids the actual intertwining operator that effects the decomposition as well as the L^2 or L^1 convergence estimates usually necessary to derive the intertwining operator. Such analytic components are often needed in applications of the direct integral formula—e.g., to solvability of differential operators [10].