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ON LIPSCHITZ STABILITY FOR F. D. E.

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Fozi M. Dannan and Saber Elaydi presented Lipschitz stability of O. D. E., and made a comparison between Lipschitz stability and Liapunov stability. In this paper, we will extend the concept of Lipschitz stability to the systems of functional differential equations (F.D.E.), and give some criteria via Liapunov's second method.

1. Definitions. We consider the system

$$x'(t) = f(t, x_t), \tag{1.1}$$

where $x \in R^n$, $f\colon R \times C([-r, 0], R^n) \mapsto R^n$, $f(t, 0) = 0$, f is continuous, $x_t = x(t+\theta)$, $\theta \in [-r, 0]$, $r > 0$. The initial value condition associated with (1.1) is

$$x(\theta) = \phi(\theta), \quad \theta \in [-r, 0], \quad \phi(\theta) \in C([-r, 0], R^n). \tag{1.2}$$

Set $\|\phi\| = \sup_{\theta\in[-r,0]} |\phi(\theta)|$, where $|\cdot|$ is a norm in R^n. We always assume that the solution of (1.1) with (1.2) is existent and unique.

DEFINITION 1. For the solution $x(t)$ of (1.1) through (t_0, ϕ), (see [2, p. 38]), $(t_0, \phi) \in R^+ \times C([-r, 0], R^n)$, $R^+ \stackrel{\text{def}}{=} [0, +\infty)$, if there exists a constant $\delta > 0$, which is independent of t_0, and another constant $M = M(\delta) > 0$, such that

$$|x_t| \le M\|\phi\|, \quad \text{for } t \ge t_0, \text{ and } \|\phi\| < \delta, \tag{1.3}$$

then the zero solution of (1.1) is said to be Lipschitz uniformly stable. This is denoted by $(1.1) \in \text{Lip. U. S.}$

DEFINITION 2. If in Definition 1, δ is allowed to be $+\infty$, then the zero solution of (1.1) is said to be Lipschitz globally uniformly stable. This is denoted by $(1.1) \in \text{Lip. G. U. S.}$

Obviously, if $r = 0$, each definition above reduces to a definition for O.D.E.

If on $[t_0, T]$, where T is large enough, the solution of (1.1) through (t_0, ϕ) satisfies Definitions 1 and 2, it is said to be Lipschitz uniformly or globally uniformly stable on the large interval $[t_0, T]$.