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ARENS REGULARITY AND DISCRETE GROUPS

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Let G be a locally compact group. Let $A_p(G)$ be the Herz algebra of G associated with $1 < p < \infty$. We show that if $A_p(G)$ is Arens regular, then G is discrete. We also exhibit a number of sufficient conditions for such a group to be finite.

1. Introduction. Let G be a locally compact group. For $1 < p < \infty$, let $A_p(G)$ denote the linear subspace of $C_0(G)$ consisting of all functions of the form $u(x) = \sum_{i=1}^{\infty}(f_i * \tilde{g}_i)^{\vee}$, where $f_i \in L_p(G)$, $g_i \in L_q(G)$, $\frac{1}{p} + \frac{1}{q} = 1$, $\sum_{i=1}^{\infty} \|f\|_p \|g\|_q < \infty$, $f^{\vee}(x) = f(x^{-1})$ and $\tilde{f}(x) = \overline{f(x^{-1})}$. $A_p(G)$ is a commutative Banach algebra with respect to pointwise multiplication and the norm

$$\|u\|_{A_p(G)} = \inf\left\{\sum_{i=1}^{\infty} \|f_i\|_p \|g_i\|_q \middle| u(x) = \sum_{i=1}^{\infty}(f_i * \tilde{g}_i)^{\vee}\right\}.$$

When $p = 2$, $A_2(G)$ is the Fourier algebra of G as introduced by Eymard in [7]. For general p, the algebras $A_p(G)$ were introduced and first studied by Herz [**13**].

In this paper we will study the structure of the second dual $A_p(G)^{**}$ as a Banach algebra with respect to the two Arens products. In particular, we will show that if $A_p(G)$ is Arens regular, then G is discrete. When $p = 2$, we show that for a large class of groups, Arens regularity will imply finiteness.

2. Preliminaries. Let G be a locally compact group with a fixed left Haar measure λ. For $1 \leq p \leq \infty$, let $L_p(G)$ be the usual Banach space of equivalence classes of p-integrable (or essentially bounded) functions on G. The algebras $A_p(G)$ for $1 < p < \infty$ will be as defined in §1. When $p = 2$ we will write $A(G)$ for $A_2(G)$.

For $1 < p < \infty$, let $PF_p(G)$ and $PM_p(G)$ denote the closure of $L_1(G)$, considered as an algebra of convolution operators on $L_p(G)$, with respect to the norm topology and the weak operator topology respectively in $\mathscr{B}(L_p(G))$, the bounded operators on $L_p(G)$. The space $PM_p(G)$ can be identified with the dual of $A_p(G)$ for each $1 < p < \infty$ [see **19**, p. 94].