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ON REGULAR COVERINGS OF 3-MANIFOLDS BY HOMOLOGY 3-SPHERES

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We study homology 3-spheres $\widetilde{M}$ that admit fixed point free actions by a finite group G. If G also admits a fixed point free orthogonal action on S^3 and if certain projective $Z[G]$-modules satisfy a cancellation property we show that the regular covering $\widetilde{M} \to \widetilde{M}/G$ is induced from a standard regular covering $S^3 \to S^3/G$ by means of a map $f\colon \widetilde{M}/G \to S^3/G$ whose degree is relatively prime to the order of G (Theorem 1). We also completely characterize those regular coverings $\widetilde{M} \to M$ where M is Seifert fibered (§4). Finally, starting with any given regular covering $\widetilde{M}_0 \to M_0$ with group of covering transformations G, M_0 irreducible, and $\widetilde{M}_0$ a homology 3-sphere, we show how to construct another regular covering $\widetilde{M} \to M$ with $\widetilde{M}$ a homology 3-sphere and the same group G of covering transformations, with M sufficiently large, M and M_0 not homotopy equivalent, and a degree 1 map $f\colon M \to M_0$ that induces the regular covering $\widetilde{M} \to M$ from the regular covering $\widetilde{M}_0 \to M_0$.

1. Introduction. It is a classical result that the finite groups that admit a fixed point free orthogonal action on the 3-sphere S^3 are exactly the groups of the following four classes (see **[ST]** or **[Mi1]**):

(I) The binary polyhedral groups, that is, the binary dihedral groups

$$Q_{4n} = \{x, y; x^2 = (xy)^2 = y^n\}, \qquad n \geq 2;$$

the binary tetrahedral group

$$T_{24} = \{x, y; x^2 = (xy)^3 = y^3, x^4 = 1\};$$

the binary octahedral group

$$O_{48} = \{x, y; x^2 = (xy)^3 = y^4, x^4 = 1\};$$

the binary icosahedral group

$$I_{120} = \{x, y; x^2 = (xy)^3 = y^5, x^4 = 1\}.$$

(II) The groups

$$D(2^k, 2l+1) = \{x, y; x^{2^k} = 1, y^{2l+1} = 1, xyx^{-1} = y^{-1}\}, \qquad k \geq 3, l \geq 1.$$