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NOTE ON THE DISCRIMINANT OF CERTAIN CYCLOTOMIC PERIOD POLYNOMIALS

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In this note we determine the discriminant of the period polynomial for certain binomial periods in the nth cyclotomic field, where $n = pq$ and p, q are distinct, odd primes.

1. Introduction. The binomial cyclotomic periods η_j for an odd prime p are defined as

$$\eta_j = \theta_j = \zeta^j + \zeta^{-j}, \qquad 1 \le j \le \frac{p-1}{2}, \tag{1}$$

where $\zeta = \zeta_p$ denotes a primitive pth root of unity, which in the standard Gaussian notation is the case where $e = (p-1)/2$ and $f = 2$. The associated period polynomial is defined to be

$$\Psi_p(x) = \prod_{j=1}^{\frac{p-1}{2}} (x - \theta_j) = x^e + a_1 x^{e-1} + \cdots + a_e, \tag{2}$$

where

$$a_k = (-1)^{[\frac{k}{2}]} \binom{e - [\frac{k+1}{2}]}{[\frac{k}{2}]}, \qquad 1 \le k \le e.$$

(Here we use θ_j when η_j is real. The square bracket is the greatest integer function.)

The formula for the discriminant of this polynomial is

$$D(\Psi_p(x)) = p^{\frac{p-3}{2}}. \tag{3}$$

It is also useful to introduce the polynomial

$$\Psi_p^{(m)}(x) = \prod_{j=1}^{\frac{p-1}{2}} (x - \theta_j^m), \qquad m \ge 0. \tag{4}$$

(See [**4**] for references to the above.)

In [**4**], the Lehmers investigated the period polynomial for another species of binomial period, the case where $n = pq$, p and q being distinct, odd primes. As in the prime case, $\zeta = \zeta_n$ denotes a primitive nth root of unity and $f = 2$. Here $e = \phi(n)/2 = (p-1)(q-1)/2$.