PACIFIC JOURNAL OF MATHEMATICS
Vol. 152, No. 2, 1992

PULLING BACK BUNDLES

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Let D be an ample divisor on a smooth projective algebraic variety X. We will define the notion of a vector bundle $\mathscr{W}$ on X to be strongly stable with respect to D. If X has characteristic zero this definition is the same as the usual definition of stability. In general it implies stability.

Let $f\colon Y \to X$ be a finite morphism. Then we have the bundle $f^*\mathscr{W}$ on Y which has the ample divisor $f^{-1}D$. If $\mathscr{W}$ is stable with respect to D, we will prove

THEOREM 1 (Characteristic zero). *$f^*\mathscr{W}$ is the direct sum of stable bundles of the same slope with respect to $f^{-1}D$, i.e. $f^*\mathscr{W}$ is poly-stable.*

Consider the special case of a finite morphism $f\colon \mathbb{P}^n \to \mathbb{P}^n$. For instance f is given by raising the homogeneous coordinates to the kth power. Then we have an essentially unique choice of D and $f^{-1}D$. Our result is a strong version of the above problem. When rank $\mathscr{W} = 2$ this is due to Barth [5].

THEOREM 2. *If $\mathscr{W}$ is a strongly stable bundle on $\mathbb{P}^n$, then $f^*\mathscr{W}$ is strongly stable.*

By Theorem 1 in characteristic zero we need only see that $f^*\mathscr{W}$ is indecomposable. One may apply this in particular to the Mumford-Horrocks' bundle on $\mathbb{P}^4$ and thereby produce many other rank two bundles on $\mathbb{P}^4$ with larger Chern classes. See [6].

1. Stability and strong stability. Let D be an ample divisor on a smooth projective variety X. Let $\mathscr{W}$ be a torsion-free coherent sheaf on X. The slope $\mu(\mathscr{W}) = \deg\mathscr{W}/\operatorname{rank}\mathscr{W}$ where $\deg\mathscr{W} = [c_1(\mathscr{W})\cdot D^{\dim X-1}]$.

Then $\mathscr{W}$ is stable with respect to D if $\mu(\mathscr{F}) < \mu(\mathscr{W})$ for all non-zero coherent subsheaves $\mathscr{F} \subsetneq \mathscr{W}$.

For strong stability we will assume that $\mathscr{W}$ is locally free. When $\mathscr{W}$ is strongly free if for all $0 < i < \operatorname{rank}\mathscr{W}$, $\Gamma(X, \mathscr{L}^{\otimes -1} \otimes \bigwedge^i \mathscr{W}) = 0$ for all invertible sheaves $\mathscr{L}$ on X such that $\deg\mathscr{L} \geq i\mu(\mathscr{W})$.