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BOUNDARY BEHAVIOR OF A CONFORMAL MAPPING

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Let D be a simply connected plane domain, not the whole plane. Let R^* denote those accessible boundary points of D such that D twists violently about them; that is, if $\alpha \in R^*$ and $w(\alpha)$ denotes its complex coordinate, then

$$\liminf_{\substack{w \to \alpha \\ w \in D}} \arg(w - w(\alpha)) = -\infty \quad \textbf{and}$$
$$\limsup_{\substack{w \to \alpha \\ w \in D}} \arg(w - w(\alpha)) = +\infty,$$

where $\arg(w - w(\alpha))$ is defined and continuous in D. We show that if a certain geometric condition holds at each point of a set $W^* \subset R^*$, then W^* is a D-conformal null set. Let L_ν denote the ray with terminal point $w(\alpha)$, $\alpha \in R^*$, having inclination ν, $0 \le \nu < 2\pi$. Let m denote Lebesgue measure on L_ν and set

$$u(\nu) = \limsup_{r \to 0} \frac{m((L_\nu \cap D) \cap (w(\alpha), w(\alpha) + re^{i\nu}))}{r}.$$

Let $W^* = \{\alpha \in R^* :$ there exists L_{ν_i}, $i = 1, 2, 3$, at $w(\alpha)$ such that $|\nu_i - \nu_j| = (2/3)\pi$, $1 \le i < j \le 3$, and $u(\nu_i) < 1$ for $i = 1, 2, 3\}$.

THEOREM. *W^* is a D-conformal null set.*

Introduction. Let D be a simply connected plane domain, not the whole plane, and let $w = f(z)$ be a one to one conformal map from the open unit disk onto D. It is well-known that for almost every θ, $0 \le \theta < 2\pi$, $f(z)$ has a finite radial limit $f(e^{i\theta})$ at $e^{i\theta}$. By [**4**, pp. 311–312] we also have for almost every θ that the image of the radius at $e^{i\theta}$ is a rectifiable curve. Thus, for almost every θ, $0 \le \theta < 2\pi$, the image of the radius at $e^{i\theta}$ determines a (ideal) rectifiably accessible boundary point α_θ of D whose complex coordinate $w(\alpha_\theta) = f(e^{i\theta})$ is finite. The set of all such α_θ is denoted by A^*. In fact, using Theorem 1 in [**2**, p. 37], Theorem 9.3 in [**4**, p. 268], and Theorem 10.9 in [**4**, p. 316], it follows that A^* is the set of all rectifiably accessible boundary points of D. On $D^* = D \cup A^*$ we define the arc-length distance l_{D^*} between two points as the infimum of the euclidean lengths of arcs that lie in D and join these two points.