

HOMEOTOPY GROUPS OF
IRREDUCIBLE 3-MANIFOLDS
WHICH MAY CONTAIN
TWO-SIDED PROJECTIVE PLANES

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A description is obtained for the homeotopy group (the group of isotopy classes of homeomorphisms) of a compact irreducible sufficiently large 3-manifold (which may contain 2-sided projective planes). It is finitely presented, and modulo finite groups is either free, $GL(3, \mathbb{Z})$, or is built up in a certain way by extensions starting from 2-manifold homeotopy groups and finitely generated abelian groups.

0. Introduction. Manifolds containing 2-sided projective planes have been a stumbling block in the study of mappings of 3-manifolds. For orientable sufficiently large 3-manifolds with incompressible boundary (now called *Haken* 3-manifolds), Waldhausen's [W] seminal work provides (except for I -bundles) an isomorphism from the homeotopy group to the group of outer automorphisms of the fundamental group that preserve the peripheral structure. To establish this isomorphism, he shows that every proper homotopy equivalence is homotopic to a homeomorphism, and that (except for I -bundles) homotopic homeomorphisms are isotopic. This program can be extended to nonorientable manifolds which do not contain 2-sided projective planes [H2], [L], but when there are 2-sided projective planes present, the manifolds are no longer aspherical and the behavior of homotopy equivalences becomes more complicated. For example, for 3-manifolds that are nontrivial connected sums and contain a 2-sided projective plane, not every self-homotopy equivalence is homotopic to a homeomorphism [H3], [H4].

Swarup [S3] surmounted some of these difficulties. He studied the class of irreducible 3-manifolds containing 2-sided projective planes. In [S3], such a manifold is said to be *sufficiently large* if it has a hierarchy (a finite sequence of cuttings along 2-sided incompressible surfaces) which ends with a collection of 3-balls and $\mathbb{P}^2 \times I$'s. Swarup shows that if there is an isomorphism of fundamental groups between two sufficiently large irreducible boundary-irreducible 3-manifolds