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BORSUK-ULAM THEOREM, FIXED POINT INDEX AND CHAIN APPROXIMATIONS FOR MAPS WITH MULTIPLICITY

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In this article we consider m-acyclic maps with respect to a field $\mathbb{F}$ and prove the existence of chain approximation for such maps. Furthermore we provide a unified approach to the Borsuk-Ulam theorem and the Bourgin-Yang generalization. Finally we prove the existence of A-systems for certain m-acyclic maps and define a fixed point index.

There are essentially two ways to handle multivalued fixed point or coincidence problems. The first one is based on homological arguments (homological method) and the second on the homotopical method where the multivalued map is approximated with single valued maps. For a survey of both methods we recommend [**3**], for single valued maps we refer to [**4**, **8**].

The homological method also splits in two directions—the considerations on the level of homology groups and chain approximation techniques. For the first one see [**3**], the second one—chain approximations of multivalued maps—has roots in the early work of L. Vietoris (see [**1**] where the Vietoris-Begle mapping theorem is proved). The chain approximation technique is used by S. Eilenberg and D. Montgomery [**10**] to prove a Lefschetz fixed point theorem for acyclic maps on compact ANR's. B. O'Neil constructed chain approximations for a more general mapping class, i.e. $(1, n)$-mappings, and also proved a Lefschetz fixed point theorem for such mappings on polyhedra [**22**]. The same technique was the main tool for developing the fixed point index with all properties (including commutativity and mod-p-property; multiplicity is proved in [**26**]) for multivalued maps of ANR's ([**9**, **11**, **25**]). The main result in [**25**] may be stated as follows: If a class of multivalued maps has arbitrarily close chain approximations, then there is a fixed point index with all properties for this class.

In this paper we consider so called m-acyclic maps with respect to a given field $\mathbb{F}$ and prove that for such mappings there exist chain