PACIFIC JOURNAL OF MATHEMATICS
Vol. 153, No. 2, 1992

A PHRAGMÉN-LINDELÖF THEOREM

X. T. LIANG AND Y. W. LU

Let Ω be an unbounded and connected domain in E^n. Consider on $\Omega \times (0, \infty)$ the parabolic equation

$$u_t - \operatorname{div} \mathbf{A}(x, t, u, \nabla u) = B(x, t, u, \nabla u).$$

Under proper conditions a theorem of Phragmén-Lindelöf type is proved for generalized solutions of the equation.

Introduction. The classical Phragmén-Lindelöf principle gives an important property of harmonic functions defined on a plane sector domain. That has been generalized not only to generalized solutions of quasi-linear elliptic equations in more general unbounded and connected domains (see **[1]**–**[5]**), but also to the ones of quasi-linear parabolic equations in divergence form which have their principal parts only **[6]**. In this paper the result is extended to generalized solutions of the equation (1). We prove the result by an argument based on the technique of Moser [7] and Ladyženskaja-Ural′ceva **[8]**. We have not seen any reference discussing such behavior for solutions of parabolic equations except **[6]** where the simpler situation of the equation (1), namely $B \equiv 0$, is considered.

The paper is organized as follows. In §1 the main result is mentioned and in §2 several lemmata are given as preliminaries. Finally, a full proof of our theorem is stated in §3.

1. Main result. Let Ω be an unbounded and connected domain in the n-dimensional Euclidean space E^n. Denote by $\partial\Omega$ the boundary of Ω. On $\Omega \times (0, \infty)$ we consider the following equation:

$$u_t - \operatorname{div} \mathbf{A}(x, t, u, \nabla u) = B(x, t, u, \nabla u) \tag{1}$$

where $A(x, t, u, \xi)$ and $B(x, t, u, \xi)$ are defined on $\Omega \times (0, \infty) \times E^1 \times E^n$, continuous with respect to u and ξ for fixed x and t, measurable with respect to x and t for fixed u and ξ, and satisfying the following structural conditions:

$$\begin{aligned} \xi \cdot \mathbf{A}(x, t, u, \xi) &\geq \kappa_0 |\xi|^2, \\ |\mathbf{A}(x, t, u, \xi)| &\leq \kappa_1 |\xi|, \\ |B(x, t, u, \xi)| &\leq b(x, t)|\xi|, \end{aligned} \tag{2}$$