

THE PERIOD MATRIX OF BRING'S CURVE

GONZALO RIERA AND RUBÍ E. RODRÍGUEZ

In genus four there is only one Riemann surface admitting the symmetric group of order five as group of automorphisms: we compute its Riemann matrix. On the other hand, we show that there is a one complex parameter family of Jacobians admitting the same group of automorphisms and using the Schottky relation we give a non-trivial equation vanishing exactly on the matrix of the surface.

1. Introduction. When studying the general equation of degree five, Bring constructed a curve of genus four admitting the symmetric group \mathcal{S}_5 as a group of automorphisms. Namely he considered the equations

$$B: \sum_{i=1}^5 x_i = 0, \quad \sum_{i=1}^5 x_i^2 = 0, \quad \sum_{i=1}^5 x_i^3 = 0$$

in homogeneous coordinates in \mathbb{P}_4 where the group acts by permutations of the coordinates.

As this is an example of a curve with a maximal group of automorphisms in genus 4 just as Klein's curve is in genus 3, cf. [7]–[8], or as Fermat's curves are in other genus, cf. [5], one can ask whether this group can help in computing the Riemann matrix of the curve. The answer however involves considerably more difficulty than in the previous known examples and comes from the fact, as we will show, that there is a one complex parameter family of matrices in Siegel's space fixed by a group in $\mathrm{Sp}(8, \mathbb{Z})$ isomorphic to \mathcal{S}_5 . To decide which one of these matrices effectively corresponds to Bring's curve involves a transcendental equation equivalent to the vanishing of Schottky's relation and this provides a direct way of proving that this relation is not trivial (cf. [1]).

The methods we use are novel in the sense that they stem from the universal cover of B and not from this curve viewed as a covering of \mathbb{P}_1 , and are readily generalized to higher genus and to the study, to be pursued elsewhere, of families of surfaces. We would like to thank Professors A. Beauville and H. Clemens for their kind interest in this work.

2. The universal cover of Bring's curve. If $f: B \rightarrow B$ is an automorphism of a compact Riemann surface into itself, it induces an integral