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SIMPLE LOCAL TRACE FORMULAS FOR UNRAMIFIED p-ADIC GROUPS

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Let G be a connected unramified semi-simple group over a p-adic field F. In this note, we compute a (Macdonald-)Plancherel-type formula: $\int_{G(F)\times G(F)} f(h)\phi(g^{-1}hg)\,dg\,dh = \int f^\vee(\chi)I(\chi, \phi)\,d\mu(\chi)$. Here f is a spherical function, $f^\vee$ is its Satake transform, and ϕ is a smooth function supported on the elliptic set. For this, we use the Geometrical Lemma of Bernstein and Zelevinsky, Macdonald's Plancherel formula, Macdonald's formula for the spherical function, results of Casselman on intertwining operators of the unramified series, and a combinatorial lemma of Arthur. This derivation follows a procedure of Waldspurger rather closely, where the case of GL(n) was worked out in detail. We may rewrite this formula as $\int_{G(F)} f(g^{-1}\gamma g)\,dg = \int f^\vee(\chi)I(\chi, \gamma)\,d\mu(\chi)$, for γ elliptic regular in $G(F)$ and f spherical. Here $I(\chi, \gamma)$ is a distribution on the support of the Plancherel measure (regarded as a compact complex analytic variety).

Introduction. Let G be a connected unramified semi-simple group over a p-adic field F, let G_{reg} denote the subset of regular elements of $G(F)$, and let G_{ell} denote the subset of elliptic regular elements of $G(F)$. Let $C_c^\infty(G(F))$ denote the algebra of locally constant compactly supported functions on $G(F)$ and let $\mathscr{H}(G, K)$ denote the commutative subalgebra of spherical functions associated to a hyperspecial, good, maximally bounded subgroup K of $G(F)$.

Let $\Phi \subset C_c^\infty(G(F))$ denote the subspace of functions on $G(F)$ supported in G_{ell}. For each $\phi \in \Phi$, define

$$(0.1) \qquad T_\phi : f \mapsto \int_{G(F)\times G(F)} f(h)\phi(g^{-1}hg)\,dg\,dh.$$

It is not hard to show that T_ϕ defines an "elliptic" invariant distribution in $C_c^\infty(G(F))'$ with compactly generated support (that is $\operatorname{supp} T_\phi \subset C^G$, where $C \subset G(F)$ is compact and C^G denotes the set of $G(F)$-conjugacy classes containing an element of C). In this note, we restrict T_ϕ to $\mathscr{H}(G, K)$ and compute, in §§2–3, a (Macdonald-)Plancherel-type formula for T_ϕ:

$$(0.2) \qquad T_\phi(f) = \int f^\vee(\chi)I(\chi, \phi)\,d\mu(\chi)$$