PACIFIC JOURNAL OF MATHEMATICS
Vol. 154, No. 1, 1992

PSEUDO REGULAR ELEMENTS IN A NORMED RING

RICHARD F. ARENS

Let A be an algebra, and let f be a linear mapping of A into some normed linear space C. For a in A we will write af for the image of a under f. By abf we mean $(ab)f$. Suppose $\|abf\| \leq M\|af\| \cdot \|bf\|$ for some real M, and all a, b in A. Then we will say that f is *pseudo regular* for A.

We study mainly the case when $C = A$ and A is a commutative Banach algebra. We present some conditions which imply pseudo regularity, and some that prevent it. For example, if the non-zero elements of the spectrum of f are bounded away from zero, then f is pseudo regular. A result (5.3) in the other direction is that if $\sum_{-\infty}^{\infty} |tf(t)|dt < \infty$ for a pseudo regular element f of $L^1(\mathbb{Z})$, then the spectrum is bounded away from 0. Concerning the algebra $C^1[a, b]$, any f which has no zero in common with its derivative is pseudo regular.

2. The relation to regularity. Behavior on extension. Let A be a normed commutative algebra and let $f \in A$. One says that f is *subregular in A* if there is another commutative normed algebra B which contains A isomorphically, which has a unit 1, and in which the element corresponding to f in B has an inverse.

(2.1) PROPOSITION. *If f is subregular in A, it is pseudo regular in A.*

Proof. Let f have the inverse g in an algebra B containing A. Let a and b belong to A. Then $\|afbfg\| = \|afbfg\| \leq \|af\| \, \|bf\| \, \|g\|$, so f is pseudo regular in A.

(2.2) PROPOSITION. *Pseudo regularity does not imply subregularity.*

Here an example will suffice. Take A to be the space $C(S, \mathbb{C})$ of continuous functions on some compact Hausdorff space S with a non-trivial open-and-closed subset E. The characteristic function e of E satisfies $e^2 = e$, so it cannot have an inverse in any B. Thus e is not subregular. On the other hand, [**A&G**, Theorem 3.3, or (3.22) below] shows that e is pseudo regular.