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# ACTIONS OF FINITE GROUPS ON KNOT COMPLEMENTS

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**We examine the symmetry of the complement of a non-trivial knot $K$ in $S^3$ and obtain a classification of the actions of finite groups on the complement of a non-trivial knot in the case where the actions extend to non-fixed point free actions on the three sphere. We prove the result by showing first an extension theorem which says that any action of finite group on a non-trivial knot complement extends to an action on the three sphere and then by applying the solution of the Smith conjecture.**

Let $N(K)$ be a regular neighborhood of $K$; $m$, $l$ be a meridian and a preferred longitude of $K$ in $\partial N(K)$ respectively; $[m]$, $[l]$ be the homology classes in $H_1(\partial N(K), Z)$ represented by $m$, $l$ respectively. A knot is called a hyperbolic knot if $S^3 - K$ has a hyperbolic structure. See $[R]$, or $[B, Z]$ for the standard terminology that we use. The main results of this note are the following. Theorem 1 also follows from the recent result of Gordon and Luecke **[G, L]**. Since the proof is simple, it is included here for completeness.

THEOREM 1. *If $K$ is a hyperbolic knot, then any self-diffeomorphism of the knot complement $S^3 - \operatorname{int}(N(K))$ extends to a self-diffeomorphism of $S^3$.*

Satellite knots have property P by Gabai's work, and torus knots are also known to have property P. One obtains the following theorem.

COROLLARY 1. *Any self-diffeomorphism of a non-trivial knot complement $S^3 - N(K)$ extends to a self-diffeomorphism of $S^3$.*

THEOREM 2. *If $G$ is a finite group acting smoothly on the complement $S^3 - \operatorname{int}(N(K))$ of a non-trivial knot $K$, then the group $G$ is a cyclic or a dihedral group, and the $G$-action extends to a $G$-action on $S^3$. In particular, if $K$ is a hyperbolic knot, then $\operatorname{Out}(\pi_1(S^3 - K))$ (or what is the same $\operatorname{Isom}(S^3 - K)$) is a cyclic or a dihedral group.*

With the help of a computer, Riley **[Ri]** has calculated the