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TWO REMARKS ON POLYNOMIALS IN TWO VARIABLES

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Let X be a compactification of $\mathbf{C}^2$ such that a polynomial p can be extended to a regular mapping $\overline{p}: X \to \mathbf{CP}^1$. If generic fibers of p are irreducible, then we show that the number of reducible fibers is less than the number of horizontal components of the curve $X - \mathbf{C}^2$. If p is rational, then the restriction of $\overline{p}$ to every horizontal component except one is a one-to-one mapping.

Introduction. Let $p \in \mathbf{C}[x, y]$ be a polynomial in two complex variables. Recall that a polynomial fiber is a set

$$\Gamma_c = \{(x, y) \in \mathbf{C}^2 | p(x, y) = c\},$$

where $c \in \mathbf{C}$, i.e., every polynomial fiber is an affine algebraic curve. There is a finite set $S \subset \mathbf{C}$ such that for every $c, c' \in \mathbf{C} - S$ the fibers Γ_c and $\Gamma_{c'}$ are homeomorphic. If $c \in \mathbf{C} - S$, then the fiber is called generic (the definition of the generic fibers of a polynomial is a little different, but we can use this one according to [**LZ**]). A polynomial is called primitive if its generic fibers are connected. For every non-primitive polynomial $q(x, y)$ there exist a primitive polynomial $p(x, y)$ and a polynomial $h(z)$ in one variable such that $q(x, y) = h(p(x, y))$ [**LZ**], [**F**]. This fact reduces the study of polynomials in two variables to the case of primitive polynomials. From now on we shall restrict ourselves to primitive polynomials only. If a fiber of p is not homeomorphic to a generic fiber, it is called a degenerate fiber. A degenerate fiber can be reducible even when p is primitive, in other words this fiber can consist of more than one irreducible component. Let k be the degree of a primitive polynomial p. Then the number r of reducible fibers is less than $k - 1$ [**St**]. V. Ya. Lin knew, but never published a theorem from which a stronger statement follows: if the polynomial p has type (g, m) (i.e., its generic fibers are m-punctured Riemann surfaces of genus g), then the number of reducible fibers is less than $m - 1$, and it is easy to check that $m \leq k$. Our first aim is to improve these estimates.

Standard results of the theory of resolution of singularities guarantee the existence of a smooth compactification $\overline{X}$ of $\mathbf{C}^2$ such that the