ALMOST s-TANGENT MANIFOLDS OF HIGHER ORDER

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We introduce the notion of almost s-tangent structures of higher order by abstracting the geometric structure of the space of k-jets $J^k(\mathbb{R}\,,\,M)$. These structures are a natural extension of almost tangent structures of higher order.

1. Introduction. Almost tangent structures on even-dimensional manifolds were introduced by Clark and Bruckheimer [1] and Eliopoulos [6] around 1960 and have been investigated by many authors (see [15] and references therein). As it is well known the tangent bundle TM of a manifold M carries a canonical almost tangent structure (hence the name). This almost tangent structure plays an important role in the Lagrangian formulation of particle dynamics [15]. Crampin [2] showed that integrable almost tangent structures are relevant to study the inverse problem of Lagrangian dynamics and Cantrijn, Cariñena, Crampin and Ibort [3] proposed a geometric method of reduction of degenerate Lagrangian systems in this framework.

The notion of almost tangent structure of higher order is due to Eliopoulos [7]. An almost tangent structure of order k on a ((k+1)n)-dimensional manifold is defined by abstracting the geometric structure of the tangent bundle of order k of an n-dimensional manifold. Tangent bundles of higher order are the natural framework to develop the Lagrangian dynamics of higher order (see [14, 4]). Recently, de León, Giraldo and Rodrigues [12, 17] have obtained results similar to those of Cantrijn et al. in order to reduce degenerate Lagrangian systems of higher order in the framework of higher order integrable almost tangent structures.

In [20] Oubiña extended the notion of almost tangent structures to odd-dimensional manifolds and introduced a new type of geometric structures, the so called almost s-tangent structures, their model being the stable tangent bundle $J^1(\mathbb{R}, M) \equiv \mathbb{R} \times TM$. These structures are involved in the study of the non-autonomous Lagrangian systems, and the inverse problem of non-autonomous Lagrangian dynamics can be reformulated in terms of almost s-tangent structures [13].