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# $L^p$-FOURIER TRANSFORMS ON NILPOTENT LIE GROUPS AND SOLVABLE LIE GROUPS ACTING ON SIEGEL DOMAINS

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**We study Fourier transforms of $L^p$-functions ($1 < p \leq 2$) on nilpotent Lie groups and affine automorphism groups of Siegel domains. We get an estimate for the norm of the $L^p$-Fourier transform for certain classes of nilpotent Lie groups. For affine automorphism groups, which are nonunimodular, we give an explicit definition of $L^p$-Fourier transform, and obtain an estimate for the norm.**

**Introduction.** First of all, let us recall some known results of the $L^p$-Fourier transform on unimodular groups. For such groups, the classical Hausdorff-Young theorem was generalized by Kunze [**13**]. Following a description of Lipsman [**14**], we briefly mention the generalization. Let $G$ be a separable locally compact unimodular group of type I, and $\widehat{G}$ be the unitary dual endowed with the Mackey Borel structure. Denote by $dg$ a Haar measure on $G$, and by $\mu$ the Plancherel measure on $\widehat{G}$ associated with $dg$. That is, $\mu$ is uniquely determined by the abstract Plancherel formula; for $\varphi \in L^1(G) \cap L^2(G)$,

$$(0.1) \qquad \int_G |\varphi(g)|^2 \, dg = \int_{\widehat{G}} \operatorname{tr}(\pi(\varphi)^* \pi(\varphi)) \, d\mu(\pi),$$

where $\pi(\varphi) = \int_G \varphi(g)\pi(g)\, dg$. We consider the Fourier transform $\mathscr{P}$ to be a mapping of $L^1(G)$ to a space of $\mu$-measurable field of bounded operators on $\widehat{G}$; $(\mathscr{P}\varphi)(\pi) = \pi(\varphi)$, for $\varphi \in L^1(G)$, $\pi \in \widehat{G}$. Let $1 < p < 2$ and $q = p/(p-1)$, and for a $\mu$-measurable field of bounded operators $F$ on $\widehat{G}$, let

$$\|F\|_q = \left( \int_{\widehat{G}} \|F(\pi)\|_{C_q}^q \, d\mu(\pi) \right)^{1/q},$$

where $\|F(\pi)\|_{C_q} = (\operatorname{tr}(F(\pi)^* F(\pi))^{q/2})^{1/q}$. Denote by $L^q(\widehat{G})$ the Banach space defined by the space of measurable fields $F$ such that $\|F\|_q < \infty$ in the usual way (with norm $\|\cdot\|_q$). Then the Hausdorff-Young type inequality

$$(0.2) \qquad \|\mathscr{P}\varphi\|_q \leq \|\varphi\|_p$$