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THE EULER CLASS FOR "PIECEWISE" GROUPS

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The Euler class is a semiconjugacy invariant of a discrete group G of orientation preserving homeomorphisms of the circle. An element of the second cohomology group of G with integral coefficients, it is often difficult to calculate, but even its nonvanishing seems related to dynamical complexity of G. In this note, we consider a family of discrete groups $\Gamma_{H,S}(p, q)$ of homeomorphisms of the circle, whose definition generalizes that of piecewise linear homeomorphisms. We define an invariant with which one can verify the vanishing of the Euler class in a surprising range of cases. On the other hand, the vanishing of the invariant, together with a simple geometric condition, assures the nonvanishing of the Euler class.

The invariant has a simple "operational" definition, but can also be interpreted as an element of the fundamental group of the classifying space of a certain pseudogroup. We also apply it to the question of the existence of elements of finite order in the groups $\Gamma_{H,S}(p, q)$.

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1. Definitions and results.

1.1. *The groups.* Let H be a group of analytic, orientation preserving homeomorphisms of the real line $\mathbf{R}$, and let S be an H-invariant subset of $\mathbf{R}$. Let $p < q$ be elements of S in the same H-orbit. Let $S^1_{p,q}$ denote the closed interval $[p, q]$ with p and q identified as the basepoint. Define $\Gamma_{H,S}(p, q)$ to be the group of homeomorphisms g of $S^1_{p,q}$ such that there exist $s_i \in S$, $p = s_0 < \cdots < s_n = q$, so that the restriction of g to $[s_i, s_{i+1}]$ agrees pointwise with an element h_i of H. Thus, $\Gamma_{H,S}(p, q)$ is "the group of piecewise-H homeomorphisms of $S^1_{p,q}$, with breakpoints in S."

1.2. *The invariant.* Let $N = N(S)$ denote the normal subgroup of H generated by all elements which fix some point in S. If $p, q \in S$ are in the same H-orbit, the equivalence class in H/N of an element