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ON THE POSTULATION OF 0-DIMENSIONAL SUBSCHEMES ON A SMOOTH QUADRIC

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If X is a 0-dimensional subscheme of a smooth quadric $Q \cong \mathbf{P}^1 \times \mathbf{P}^1$ we investigate the behaviour of X with respect to the linear systems of divisors of any degree (a, b). This leads to the construction of a matrix of integers which plays the role of a Hilbert function of X; we study numerical properties of this matrix and their connection with the geometry of X. Further we relate the graded Betti numbers of a minimal free resolution of X on Q with that matrix, and give a complete description of the arithmetically Cohen-Macaulay 0-dimensional subschemes of Q.

Introduction. In the last few years the interest about 0-dimensional subschemes of $\mathbf{P}^n$ has greatly grown, so many recent papers concern a deep investigation into the Hilbert function, free resolution, Betti numbers, and defining equations for such subschemes. On the other hand there has been a good deal of work on two codimensional subschemes of $\mathbf{P}^n$; hence, points of $\mathbf{P}^2$, which have both conditions, have been intensively studied. The interest on points of $\mathbf{P}^2$ comes, also, because geometric properties of a variety can sometimes be given in terms of its generic hyperplane section; so, for studying curves of $\mathbf{P}^3$, one needs properties of 0-dimensional subschemes of $\mathbf{P}^2$. A complete list of papers on these topics seems impossible to do; so we insert in the references just a few of them, which are more familiar to us.

It seems natural to generalize this situation from one side studying 0-dimensional subschemes of any variety and in particular of surfaces, on the other side working on sections of varieties done by hypersurfaces of degree bigger than one. Therefore, a first step in this direction is to investigate 0-dimensional subschemes of a quadric $(\mathbf{P}^1 \times \mathbf{P}^1)$ with special regard to their behaviour with respect to the divisors of the quadric itself.

When one embeds the quadric Q in $\mathbf{P}^3$, any subscheme X of Q becomes a subscheme of $\mathbf{P}^3$; in that case one can relate properties of X as a subscheme of Q with those as a subscheme of $\mathbf{P}^3$.

Of course, studying subschemes of Q, the geometry of the surface Q plays a big role; in particular, the cohomology groups of Q play an