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# CONFORMAL DEFORMATIONS PRESERVING THE GAUSS MAP

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**In this work, given a conformal immersion $f: M^n \to \mathbb{R}^N$ of a Riemannian manifold $M^n$ into a euclidean space $\mathbb{R}^N$, we establish conditions for the existence of another conformal immersion $\overline{f}: M^n \to \mathbb{R}^N$ with the same Gauss map as $f$. In particular, for $n = 2$ and $N = 3$, these conditions are described by means of a partial differential equation on the principal curvatures of $f$.**

**0. Introduction.** Let $M^n$ be a connected $n$-dimensional Riemannian manifold and let $f: M^n \to \mathbb{R}^N$ be a conformal immersion. We denote by $F: M^n \to G_{n,N}$ the *Gauss map* of $f$, which assigns to each point $p \in M^n$ the $n$-dimensional tangent space $f_*(T_pM)$ in the Grassmannian $G_{n,N}$. We consider the following problem: Under what conditions does there exist another conformal immersion $\overline{f}: M^n \to \mathbb{R}^N$ such that the Gauss map of $\overline{f}$ coincides with the Gauss map of $f$, up to a congruence in $G_{n,N}$ induced by a congruence in $\mathbb{R}^N$? When this occurs we say that $\overline{f}$ is a *G-deformation of* $f$. This situation is equivalent to considering conformal immersions $f$ and $\overline{f}$ with parallel tangent spaces $f_*(T_pM)$ and $\overline{f}_*(T_pM)$ in $\mathbb{R}^N$, which we will always assume. The analogous problem for isometric immersions $f$ and $\overline{f}$ was considered by Dajczer and Gromoll [**D&G**].

In §1 we characterize our situation by means of a tensor field and a differentiable function satisfying certain conditions (see Proposition 1.5). This result will be used in §2, where we treat the above problem for $n = 2$.

For surfaces, we also consider the *oriented Gauss map* $F^*: M^2 \to G^*_{2,N}$, where now $f_*(T_pM)$ is seen as an oriented 2-plane in the oriented Grassmannian $G^*_{2,N}$. In regard to the above problem we have two different situations. The first one is when $f$ and $\overline{f}$ have the same oriented Gauss map. In this case, it was shown by Hoffman and Osserman [**H&O-2**] that either $f$ and $\overline{f}$ are minimal surfaces or $\overline{f}$ coincides with $f$ up to homothety and translation in $\mathbb{R}^N$. The other situation is when, for any local orientation in $M^2$, the oriented Gauss maps of $f$ and $\overline{f}$ differ by the orientation-reversing congruence in $G^*_{2,N}$. In this case we call $\overline{f}$ a $G^*$-*deformation* and say that