PACIFIC JOURNAL OF MATHEMATICS
Vol. 156, No. 2, 1992

DETERMINANTAL CRITERIA FOR TRANSVERSALITY OF MORPHISMS

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When is a sufficiently general member $f_t: X_t \to Y$, of a family of maps of smooth schemes, transverse to a given map from a smooth scheme Z to Y? Here we give criteria valid over any universally catenary base scheme in any characteristic. Roughly speaking, our criteria will hold whenever the subscheme where an appropriate bundle map drops rank is determinantal, in the sense that it has the smallest possible dimension.

In this article, we study the transversality of the members of a smooth family of morphisms to a fixed morphism. More precisely [2], let Y and Z be smooth schemes, and suppose given a family of schemes

$$X \xrightarrow{p} T,$$

a morphism

$$X \xrightarrow{f} Y$$

from the total space of the family to Y, and a fixed morphism

$$Z \xrightarrow{g} Y,$$

all over a general base scheme S, of arbitrary characteristic. A morphism $f|p^{-1}(t)$, from a member $p^{-1}(t)$ of the family, is said to be *transverse* to g if the fiber product $p^{-1}(t) \times_Y Z$ is smooth. In many applications, Z will be a subscheme of Y via g, and each member of the family will be a subscheme of Y via the map $f|p^{-1}(t)$. Then the map $f|p^{-1}(t)$, or, as we shall say here, the family member $p^{-1}(t)$, is transverse to g precisely when the intersection $p^{-1}(t) \cap Z$ is a smooth subscheme of Y.

For applications, it is useful to have criteria which guarantee transversality for general $t \in T$, and the goal of this paper is to describe some very general ones. Earlier work in this area, drawing on ideas of Grothendieck [2] and Zariski [8], culminates most clearly in Kleiman's paper [**4**], of 1973. This work, which triggered ours, falls clearly into two theoretical contexts, distinguished both by geometric emphasis and technical style.