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GENERALIZED HORSESHOE MAPS AND INVERSE LIMITS

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The now-classical example due to Smale, the horseshoe map, displays interesting dynamics as well as a topologically complicated attractor. In 1986 Marcy Barge showed that the full attracting sets of horseshoe maps are homeomorphic to inverse limits of the unit interval with a single bonding map. Here we extend Barge's results to a more general class of maps.

1. Introduction. In **[Ba]**, Barge describes the attracting sets of horseshoe maps as inverse limits of the unit interval with a single bonding map. Topologically these spaces are chainable continua known as Knaster continua.

In this paper we consider a more general class of maps which we will refer to as generalized horseshoe maps. We will show that the attractors of these maps are homeomorphic to inverse limits of the unit interval with a single bonding map. Both the generalized horseshoe map and the bonding map which defines the inverse limit space described above "follow a pattern" in a sense we will define in the next section. In §3 we will prove two theorems about inverse limit spaces which will be needed in the proof of the main result given in §4. In the final section of the paper we will give some examples, and show that the horseshoe maps which Barge studied in **[Ba]** are special cases of the generalized horseshoes we consider here. For basic information on attractors and inverse limits see **[S]**.

2. Preliminaries. Let I denote the unit interval and $\{f_n\}_{n=1}^{\infty}$ be a sequence of maps of I into I. Let

$$(I, f_n) = \{(x_0, x_1, \ldots) : x_n \in I \text{ and } f_n(x_{n+1}) = x_n, \ n = 1, 2, \ldots\}$$

be the inverse limit space with bonding maps f_n and topology induced by the metric

$$d((x_0, x_1, \ldots), (y_0, y_1, \ldots)) = \sum_{n=0}^{\infty} \frac{|x_n - y_n|}{2^n}.$$