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STUDYING LINKS VIA CLOSED BRAIDS VI: A NON-FINITENESS THEOREM

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Exchange moves were introduced in an earlier paper by the same authors. They take one closed n-braid representative of a link to another, and can lead to examples where there are infinitely many conjugacy classes of n-braids representing a single link type.

THEOREM 1. ***If a link type has infinitely many conjugacy classes of closed n-braid representatives, then $n \geq 4$ and the infinitely many classes divide into finitely many equivalence classes under the equivalence relation generated by exchange moves.***

This theorem is the last of the preliminary steps in the authors' program for the development of a calculus on links in S^3.

THEOREM 2. ***Choose integers $n, g \geq 1$. Then there are at most finitely many link types with braid index n and genus g.***

Introduction. This paper is the sixth in a series in which the authors study the closed braid representatives of an oriented link type $\mathscr{L}$ in oriented 3-space. The earlier papers in the series are **[B-M,I]**–**[B-M,V]**. An overall view of the program may be found in **[B-M]**. The long-range goal of the program is to classify link types, up to isotopy in oriented 3-space, using techniques based upon the theory of braids. This paper is the last of the preliminary steps on the way to so doing.

Let $\mathscr{L}$ be an oriented link in oriented 3-space, and let $\mathbf{L}$ be a closed n-braid representative of $\mathscr{L}$, with braid axis $\mathbf{A}$. If the isotopy class of $\mathbf{L}$ in $S^3 - \mathbf{A}$ has a representative which has the very special form illustrated in Figure 1 (see next page), then $\mathbf{L}$ is said to admit an *exchange move*, as illustrated in Figure 1. (The example shown there is a 4-braid; however if each strand is replaced by some number of parallel strands, it can be reinterpreted as an n-braid, for any n.) Exchange moves take closed n-braids to closed n-braids, in general changing the conjugacy class. Figure 2 (see next page) shows how n-braids which admit exchange moves may be modified to produce infinitely many closed n-braid representatives of $\mathscr{L}$. In effect, the exchange move allows one to replace the sub-braid $\mathbf{X}$