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SURFACES IN THE 3-DIMENSIONAL LORENTZ-MINKOWSKI SPACE SATISFYING $\Delta x = Ax + B$

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In this paper we locally classify the surfaces M_s^2 in the 3-dimensional Lorentz-Minkowski space $\mathbb{L}^3$ verifying the equation $\Delta x = Ax + B$, where A is an endomorphism of $\mathbb{L}^3$ and B is a constant vector.

We obtain that classification by proving that M_s^2 has constant mean curvature and in a second step we deduce M_s^2 is isoparametric.

0. Introduction. In **[FL90]** the last two authors obtain a classification of surfaces M_s^2 in the 3-dimensional Lorentz-Minkowski space satisfying the condition $\Delta H = \lambda H$, for a real constant λ, where H is the mean curvature vector field. That equation is nothing but a system of partial differential equations, so that the problems quoted in **[FL90]** can be framed in a more general situation: classify semi-Riemannian submanifolds by means of some characteristic differential equations. In this line, the technique of finite type submanifolds, created and developed by B. Y. Chen, has been shown as a fruitful tool to inquire into not only the intrinsic configuration of the submanifold, but also the extrinsic one, because the Laplacian of the isometric immersion is essentially the mean curvature vector field of the submanifold.

Following Chen's idea, Garay **[Gar88]** has obtained a characterization of connected, complete surfaces of revolution in $\mathbb{E}^3$ whose component functions in $\mathbb{E}^3$ are eigenfunctions of its Laplacian with possibly distinct eigenvalues. In a second step, in **[Gar90]**, Garay found that the only Euclidean hypersurfaces whose coordinate functions are eigenfunctions for its Laplacian are open pieces of a minimal hypersurface, a hypersphere or a generalized circular cylinder.

More recently, in **[DPV90]**, Dillen-Pas-Verstraelen pointed out that Garay's condition is not coordinate invariant as a circular cylinder in $\mathbb{E}^3$ shows. Then they study and classify the surfaces in $\mathbb{E}^3$ which satisfy $\Delta x = Ax + B$, where Δ is the Laplacian on the surface, x represents the isometric immersion in $\mathbb{E}^3$, $A \in \mathbb{E}^{3\times 3}$ and $B \in \mathbb{R}^3$.

It is well known that when the ambient space is the 3-dimensional