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A_∞ AND THE GREEN FUNCTION

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Let $G(x)$ be the Green function in a domain $\Omega \subseteq \mathbb{R}^m$ with a fixed pole, and Γ be an $(m-1)$-dimensional hyperplane. We give conditions on Ω and $\Omega \cap \Gamma$ so that $|\nabla G|$ is A_∞ with respect to the $(m-1)$-dimensional measure on $\Omega \cap \Gamma$. Certain properties of the Riemann mapping of a simply-connected domain in $\mathbb{R}^2$ are extended to the Green function of domains in $\mathbb{R}^m$.

In [3], Fernández, Heinonen and Martio have proved the following:

THEOREM A. *Let f be a conformal mapping from a simply-connected planar domain Ω onto the unit disk Δ and L be a line segment in Ω. Then $f(L)$ is a quasiconformal arc. Moreover, if L is a line segment on the boundary of a half plane contained in Ω, then $|f'| \in A_\infty(ds)$ on L with respect to the linear measure ds.*

If L is any line segment in Ω, $|f'|$ need not be in $A_\infty(ds)$ on L. In fact, Heinonen and Näkki [9] have proved the following:

THEOREM B. *Let f be a conformal mapping from a simply-connected domain Ω onto the unit disk Δ and L be a line segment in Ω. Then the following are equivalent:*
(1) *$|f'| \in A_\infty(ds)$ on L,*
(2) *$f|L$ is quasisymmetric,*
(3) *there exists a chord arc domain $D \subseteq \Omega$ so that $L \subseteq \overline{D}$,*
(4) *there exists a quasidisk $D \subseteq \Omega$ so that $L \subseteq \overline{D}$.*

Let μ and ν be two measures on $\mathbb{R}^m$ $(m \geq 2)$. Recall that μ belongs to the Muckenhoupt class $A_\infty(d\nu)$ if there exist $\alpha, \beta \in (0, 1)$ such that whenever E is a measurable subset of a cube Q,

$$(0.1) \qquad \nu(E)/\nu(Q) < \alpha \quad \text{implies} \quad \mu(E)/\mu(Q) < \beta.$$

If μ and ν have the doubling property, then $\mu \in A_\infty(d\nu)$ if and only if $\nu \in A_\infty(d\mu)$ ([2]). We say a function is in $A_\infty(d\nu)$ on L, provided that (0.1) holds with $d\mu = g\,d\nu$ for all cubes $Q \subseteq L$.

$f|L$ is quasisymmetric provided that for all $a, b, x \in L$, $|a-x| \leq |b-x|$ implies $|f(a)-f(x)| \leq c|f(b)-f(x)|$ for some constant $c > 0$.