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FREE BANACH-LIE ALGEBRAS, COUNIVERSAL BANACH-LIE GROUPS, AND MORE

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The construction of free Banach-Lie algebra over a normed space enables us to build a connected separable Banach-Lie group of which any other connected separable Banach-Lie group is a quotient. New proofs are given to the result on representability of any Banach-Lie algebra as a quotient of an enlargable Banach-Lie algebra (due to van Est and Świerczkowski) and to the result on representability of any topological group as a quotient of a group with no small subgroups (due to successive efforts of Morris and Thompson, the author, and Sipacheva and Uspenskiĭ).

1. Introduction. Over the last 50 years a number of constructions of "universal arrows" (see, e.g., [**Go**]) to the categories of topological algebraic systems have been studied. Important contributions are those by Markov [**M**], Graev [**Gr**], and Arhangel'skiĭ [**A2**] on free topological groups, Mal'cev [**Mc**] on free topological algebras, Arens and Eells [**AE**], Raĭkov [**R**], and Uspenskiĭ [**U**] on free Banach spaces and free locally convex spaces. By virtue of these constructions a first ever example of a non-normal Hausdorff topological group was obtained [**M**], and the representability of any topological group as a quotient group of a zero-dimensional group was proved [**A1**]. Here we apply the concept of a free complete normed Lie algebra to theory of topological and Lie groups. Our construction is an extension of the well-known construction of Arens-Eells [**AE**] to the case of normed Lie algebras. Our main result is that there exists a couniversal separable connected Banach-Lie group, that is, such a separable connected Banach-Lie group that any other such Banach-Lie group is its quotient Lie group. This follows from observation that any free Banach-Lie algebra is enlargable, that is, comes from an approriate Banach-Lie group. Also we give entirely new and rather transparent proofs of two earlier known results.

Cohomological technique has enabled van Est and independently Świerczkowski [**Ś2**] to prove that any Banach-Lie algebra is a quotient algebra of an enlargable Banach-Lie algebra. Here we deduce the result from enlargability of free Banach-Lie algebras.

In his book [**Ka**] Kaplansky asked whether a quotient group of a