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A STATE MODEL FOR THE MULTI-VARIABLE ALEXANDER POLYNOMIAL

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We construct a vertex type state model in Turaev's sense for the multi-variable (non-reduced) Alexander polynomial. Our model is a colored version of the 6-vertex free fermion model. To show the correspondence of our model and the multi-variable Alexander polynomial, we introduce colored braid groups and their Magnus representations. By using this model, a new set of axioms for the multi-variable Alexander polynomial is obtained.

1. Introduction. In [1], the Jones polynomial V in [9] and its higher spin versions are directly constructed from some solutions of Yang-Baxter equations. Let P be the HOMFLY polynomial in [5], [16] and F be the Kauffman polynomial in [12]. Then these invariants are both two-variable extensions of the Jones polynomial V. In [19], Turaev constructs P and F from vertex type state models. Turaev introduced an enhanced Yang-Baxter operator, from which we get an invariant of links. He constructed enhanced Yang-Baxter operators from the R-matrices in [7] and showed that the related invariants are specializations of P and F. But this family does not contain the Alexander polynomial, which is the most famous link invariant. Deguchi and Akutsu [4] propose enhanced Yang-Baxter operators associated with a family of link invariants, which includes Turaev's family corresponding to P and also includes the reduced Alexander polynomial. We construct an enhanced Yang-Baxter operator for the Conway potential function ∇. The potential function ∇ is a version of the non-reduced Alexander polynomial. As is shown in [6], ∇ of a link is defined uniquely as a Laurent polynomial in variables associated with the connected components of the link. Kauffman gives an interpretation of the multi-variable Alexander polynomial by using a state model in §6 of [11]. In his model, there is no corresponding model in statistical mechanics. On the other hand, as is shown in Remark 2.4, our model comes from a solution of the Yang-Baxter equation, which assures the solvability of a lattice model in statistical mechanics.

In §2, we introduce an enhanced colored Yang-Baxter operator. This operator was introduced by Turaev [19] for non-colored links. From