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# ASYMPTOTIC EXPANSION AT A CORNER FOR THE CAPILLARY PROBLEM: THE SINGULAR CASE

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**Consider the solution of the capillary surface equation near a corner of the base domain. It is shown that there exists an asymptotic expansion of the height rise of the surface in a wedge when $\alpha+\gamma < \pi/2$, where $2\alpha$ is the corner angle and $0 \le \gamma < \pi/2$ the contact angle between the surface and the container wall. The asymptotic does not depend on the particular solution considered. In particular, the leading singularity which was discovered by Concus and Finn is equal to the solution up to $O(r^3)$.**

**1. Introduction.** We consider the non-parametric capillary problem in the presence of gravity. One seeks a surface $S$: $u = u(x)$, defined over a base domain $\Omega \subset \mathbb{R}^2$, such that $S$ meets vertical cylinder walls over the boundary $\partial\Omega$ in a prescribed constant angle $\gamma$. This problem leads to the equations, see Finn [**6**],

$$\operatorname{div} Tu = \kappa u \quad \text{in } \Omega, \tag{1.1}$$

$$\nu \cdot Tu = \cos\gamma \quad \text{on the smooth parts of } \partial\Omega, \tag{1.2}$$

where

$$Tu = \frac{Du}{\sqrt{1+|Du|^2}},$$

$\kappa = \text{const} > 0$ and $\nu$ is the exterior unit normal on the smooth parts of $\partial\Omega$.

Let $x = 0$ be a corner of $\Omega$ with the interior angle $2\alpha$ satisfying

$$0 < 2\alpha < \pi. \tag{1.3}$$

We assume that the corner is bounded by lines near $x = 0$ and that $\Omega_{R_0} = \Omega \cap B_{R_0}$ coincides with the circular sector

$$\{x \in \mathbb{R}^2 | x_1 \tan\alpha \ge |x_2|\} \cap B_{R_0}$$

for a sufficiently small $R_0 > 0$, where

$$B_R = \{x \in \mathbb{R}^2 | x_1^2 + x_2^2 < R\}.$$

Furthermore, we assume that the contact angle satisfies

$$0 \le \gamma < \pi/2. \tag{1.4}$$