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CONTRACTIVE ZERO-DIVISORS IN BERGMAN SPACES

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Generalizing a recent result of H. Hedenmalm for $p = 2$, a contractive zero-divisor is found in the Bergman space A^p over the unit disk for $1 \leq p < \infty$. This is a function $G \in A^p$ with $\|G\|_p = 1$ and a prescribed zero-set $\{\zeta_j\}$, uniquely determined by the contractive property $\|f/G\|_p \leq \|f\|_p$ for all $f \in A^p$ which vanish on $\{\zeta_j\}$. The proof uses the positivity of the biharmonic Green function of the disk. For a finite zero-set, the canonical divisor G is represented explicitly in terms of the Bergman kernel of a certain weighted A^2 space. It is then shown that G has an analytic continuation to a larger disk.

0. Introduction. It is well known that the zero-sets $\{\zeta_j\}$ of functions f in the Hardy space H^p are characterized by the Blaschke condition $\sum(1 - |\zeta_j|) < \infty$, which guarantees the convergence of the Blaschke product

$$B(z) = \prod_j b(z, \zeta_j), \quad \text{where } b(z, \zeta) = \frac{|\zeta|}{\zeta}\frac{\zeta - z}{1 - \overline{\zeta}z}$$

if $\zeta \neq 0$, and $b(z, 0) = z$. A basic theorem of F. Riesz (see [**3**], Ch. 2) asserts that f/B is a nonvanishing function in H^p with norm equal to that of f. This simple fact plays an important role in the theory of H^p spaces, and it would be desirable to find an analogue for the Bergman spaces. We shall see in this paper that while *isometric* zero-divisors are not available in the Bergman spaces, there is an essentially unique contractive divisor of unit norm associated with every zero-set.

A function f analytic in the unit disk $\mathbb{D}$ is said to belong to the *Bergman space* A^p, where $0 < p < \infty$, if

$$\|f\|_p^p = \iint_{\mathbb{D}} |f(z)|^p \, d\sigma < \infty.$$

Here $d\sigma$ denotes the normalized element of area: $d\sigma = \frac{1}{\pi} dx\, dy$. For $1 \leq p < \infty$, the *Bergman kernel* $K(z, \zeta) = (1 - \overline{\zeta}z)^{-2}$ has the reproducing property

$$f(z) = \iint_{\mathbb{D}} f(\zeta) K(z, \zeta) \, d\sigma, \qquad z \in \mathbb{D},$$