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# ON THE ANALYTIC REFLECTION OF A MINIMAL SURFACE

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**For a long time it has been known that in a Euclidean space one can reflect a minimal surface across a part of its boundary if the boundary contains a line segment, or if the minimal surface meets a plane orthogonally along the boundary. The proof of this fact makes use of H. A. Schwarz's reflection principle for harmonic functions.**

**In this paper we show that a minimal surface, as a conformal and harmonic map from a Riemann surface into $\mathbf{R}^3$, can also be reflected analytically if it meets a plane at a constant angle.**

THEOREM 1. *Let $\Sigma \subset \mathbf{R}^3$ be a minimal surface with nonempty boundary $\partial\Sigma$ and let $\Pi$ be a plane. Suppose that $L \subset \Sigma \cap \Pi$ is a $C^1$ curve, $\Sigma$ is $C^1$ along $L$, and at all points of $L$ the tangent plane to $\Sigma$ makes a fixed angle $0 < \theta < 90°$ with $\Pi$. Then $\Sigma$ can be analytically extended across $L$ to a minimal surface $\overline{\Sigma}$ satisfying the following properties:*

(i) *$\overline{\Sigma} = \Sigma \cup \Sigma^*$, where $\Sigma^*$ is the set of all images $p^*$ of $p \in \Sigma$ under an analytic reflection map $*$.*

(ii) *$p$ and $p^*$ are separated by $\Pi$ in such a way that*

$$\text{dist}(p, \Pi) = \text{dist}(p^*, \Pi).$$

(iii) *The Gauss map $g: \overline{\Sigma} \to \mathbf{C}$ satisfies*

$$\overline{g(p)} \cdot g(p^*) = \left(\tan\frac{\theta}{2}\right)^{-2}.$$

(iv) *$p^* \in \Sigma^*$ is a branch point (geometric) if and only if $p \in \Sigma$ is.*

(v) *The map $*$ is a single-valued immersion if $\Sigma$ is simply connected and $L$ is connected, or $\Sigma$ is doubly connected and $L$ is closed.*

(vi) *If $*$ is single-valued, then $\Sigma^*$ has finite total curvature if and only if $\Sigma$ does.*

(vii) *If $\partial\Sigma = L$, then $\overline{\Sigma}$ is complete.*

*Proof.* Let $x, y, z$ be coordinates of $\mathbf{R}^3$ such that $\Pi = \{(x, y, z): z = 0\}$. Since $x, y, z$ are harmonic functions on the minimal surface $\Sigma$, one can find conjugate harmonic (possibly multiple-valued)