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AN ANALYTIC FAMILY OF UNIFORMLY BOUNDED REPRESENTATIONS OF A FREE PRODUCT OF DISCRETE GROUPS

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We construct for each $|z| < 1$ a uniformly bounded representation π_z of a free product group. The correspondence $z \mapsto \pi_z$ is proved to be analytic. The representations are irreducible if the free product factors are infinite groups. On free groups they have as coefficients block radial functions—gives thus a new series of representations. They can be made unitary iff $z \in (-\frac{1}{N-1}, 1)$.

This paper is devoted to the construction of a family $\{\pi_z : |z| < 1\}$ of uniformly bounded representations of a free product of infinite groups. The construction is based on the ideas of Pytlik and Szwarc, who considered free groups on countably many generators. We have investigated a family of block radial functions discovered by W. Młotkowski. The functions were defined as follows: for $|z| < 1$,

$$\varphi_z(x) = \begin{cases} 1 & \text{if } x = e, \\ \frac{(N-1)z+1}{Nz} z^{\|x\|} & \text{if } x \neq e. \end{cases}$$

Each of these functions turns out to be a matrix coefficient of one of our representations $\{A_z : |z| < 1\}$, namely:

$$\varphi_z(x) = \langle A_z(x)\xi, \xi\rangle$$

where ξ is the common cyclic vector. The constructed representations will be shown to be irreducible, except when $z = 0$ or $z = -\frac{1}{N-1}$, which independently follows from Szwarc's general theorem on the family $\{\varphi_z : |z| < 1\}$ (see **[Sz.2]** Theorem). In the two exceptional cases $z = 0$ and $z = -\frac{1}{N-1}$ we identify the representations with the regular and the quasi-regular representation, respectively.

Next we consider the problem of whether some of the representations $\{A_z\}$ can be made unitary. For this purpose we introduce a family of operators $\{V_z : z \in \Omega\}$ where $\Omega = \{|z| < 1\} \setminus (-1, -\frac{1}{N-1}]$ and intertwine each representation by a proper V_z. In this way we get (Theorem 11) an analytic family of uniformly bounded representations $\{\pi_z : z \in \Omega\}$ which are unitary if and only if $z \in (-\frac{1}{N-1}, 1)$. All the representations are irreducible if the free product factors are