PACIFIC JOURNAL OF MATHEMATICS
Vol. 157, No. 2, 1993

ON SOME EXPLICIT FORMULAS IN THE THEORY OF WEIL REPRESENTATION

R. RANGA RAO

The object of this paper is to derive some explicit formulae concerning the Weil representation that allow us to define this projective representation in a unique manner for each choice of symplectic basis.

Let F be a self-dual locally compact field of char $\neq 2$ and X a symplectic vector space over F. Let V, V^* be two transversal Lagrangian subspaces. Then a classical construction due to Shale-Segal-Weil gives a projective representation of the symplectic group $\mathrm{Sp}(X)$ in the Schwartz-space of V. The operators $\xi(\sigma)$ corresponding to each $\sigma \in \mathrm{Sp}(X)$ are determined uniquely only up to a scalar multiple. The starting point of this paper is an explicit integral formula for these operators $\xi(\sigma)$, valid for all $\sigma \in \mathrm{Sp}(X)$. In fact (see Lemma 3.2) we have for each $\sigma \in \mathrm{Sp}(X)$

$$\xi(\sigma)\varphi : x \to \int_{V^*/\ker\gamma} f_\sigma(x, x^*)\varphi(x\alpha + x^*\gamma)\, d\mu_\sigma$$

where μ_σ is a Haar measure on $V^*/\ker\gamma$, and f_σ is the character of second degree on X, associated to σ. Here $\sigma = \left[\begin{smallmatrix}\alpha & \beta \\ \gamma & \delta\end{smallmatrix}\right]$ is the matrix representation of σ in the decomposition $X = V + V^*$. This formula is known and already present in Weil's paper when $\gamma = 0$ or when γ is an isomorphism. The extension of its validity for all σ enables us to show that it is possible to define the projective representation in a unique way for each choice of symplectic basis. Let $e_1, \dots, e_n$, $e_1^*, \dots, e_n^*$ be a symplectic basis of X such that $e_1, \dots, e_n$ $(e_1^*, \dots, e_n^*)$ is a basis of V (V^*). Let W be the finite subgroup of $\mathrm{Sp}(X)$ consisting of all σ such that $\{e_i, e_i^*\}\sigma \subseteq \{\pm e_i, \pm e_i^*\}$ for each i. Then one has the well-known Bruhat decomposition $\mathrm{Sp}(X) = PWP$, where P is the stabilizer of V^*. Then it is shown that it is possible to make consistent choices of the Haar measures μ_σ so that (1) $\xi(p_1\sigma p_2) = \xi(p_1)\xi(\sigma)\xi(p_2)$ for all $p_1, p_2 \in P$ and (2) $\xi(\sigma_1\sigma_2) = \xi(\sigma_1)\xi(\sigma_2)$ for all $\sigma_1, \sigma_2 \in W$. Moreover all such are determined. Among these there is one choice $\sigma \to r(\sigma)$ called the standard model which in addition satisfies nonnegativity properties similar to those of the Fourier Transform. All