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ON THE INCIDENCE CYCLES OF A CURVE: SOME GEOMETRIC INTERPRETATIONS

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In this paper, we note that the incidence cycles of a seminormal curve X intervene in the calculation of the arithmetic genus $p_a(X)$, of the algebraic fundamental group $\pi_1^{\mathrm{alg}}(X)$ and of the Picard group $\mathrm{Pic}(X)$ of X. Really we do not consider only seminormal curves, but more generally varieties obtained from a smooth variety by glueing a finite set of points.

0. Introduction. By a curve we mean a dimension 1 quasi-projective scheme over an algebraically closed field k.

Let X be a connected reduced seminormal curve (see [**T**], [**P**] and [**D**] for the definition and the geometric meaning of seminormality).

Let $X_1, \dots, X_n$ be the irreducible components of X; the normalization $\overline{X}$ of X is isomorphic to the disjoint union $\bigsqcup_{i=1}^{n} \overline{X}_i$ of the normalizations $\overline{X}_i$ of the curves X_i. Let $\pi\colon \overline{X} \to X$ denote the normalization morphism.

Let $P_1, \dots, P_m$ be the singular points of X and let $x_1, \dots, x_M$ be the branches of X ($x \in \overline{X}$ is a branch of X over a singular point P of X if $x \in \pi^{-1}(P)$).

We define $\nu(X) = M - m - n + 1$. In [**R**] one can find a geometric characterization of the number $\nu(X)$ in terms of the incidence cycles of X. One associates to the curve X the graph Γ whose vertices are $P_1, \dots, P_m, X_1, \dots, X_n$ and whose edges represent the M branches of X in this way: if x_r is a branch over P_i and $x_r \in \overline{X}_j$, an edge joining P_i and X_j is constructed. Any cycle of the graph Γ associated to X is said to be an *incidence cycle* of X.

In [**R**] it is proved that the graph Γ associated to X is connected, the number of the independent cycles of Γ is $\nu(X)$ and Γ contains cycles if and only if X satisfies one of the following conditions:

(a) an irreducible component of X is not locally unibranch,
(b) two irreducible components of X meet in more than one point,
(c) X contains polygons.

In the present paper we'll consider more generally a class of varieties X of dimension $r \geq 1$ and we'll see that the number $\nu(X)$