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CONCORDANCES OF METRICS OF POSITIVE SCALAR CURVATURE

PAWEŁ GAJER

Spaces of metrics of positive scalar curvature are studied modulo a concordance relation. It is shown that the set of concordance classes of metrics with positive scalar curvature on a closed manifold of dimension ≥ 6 depends only on the dimension, the first Stiefel-Whitney class of the manifold, and the cokernel of a homomorphism $\pi_2(M^n) \to \widetilde{KO}(S^2)$. In addition, for every nonnegative integer i the ith concordance group of metrics of positive scalar curvature is defined and it is shown that for a spin manifold the group is nontrivial when $n + i = 4k + 3$, $8k$, $8k + 1$, $k \geq 1$.

Two metrics g_0 and g_1 with positive scalar curvature on M^n are *concordant*, written $g_0 \cong g_1$, if there is a metric g of positive scalar curvature on $M^n \times [0, 1]$ such that $g|_{M^n \times \{i\}} = g_i$ for $i = 0, 1$, and g is a product near $M^n \times \partial[0, 1]$. It is easy to see that concordance is an equivalence relation. The set of its equivalence classes on the space $\mathbb{PSC}(M^n)$ of metrics of positive scalar curvature on M^n will be denoted by $\pi_0^c(\mathbb{PSC}(M^n))$. For $n \geq 3$ the connected sum operation induces a group structure on $\pi_0^c(\mathbb{PSC}(S^n))$ **[G]**. It will be proved that for every simply-connected spin manifold M^n of dimension $n \geq 6$ the group acts freely and transitively on $\pi_0^c(\mathbb{PSC}(M^n))$. This is a special case of the following result.

THEOREM 2.1. *Let M^n be a closed manifold of dimension $n \geq 6$. There exists a group depending only on the dimension, the first Stiefel-Whitney class of M^n, and the cokernel of the homomorphism*

$$\pi_2(M^n) \to \widetilde{KO}(S^2) \quad \text{given by} \quad [\varphi] \mapsto [\varphi^* TM^n]$$

that acts freely and transitively on the set $\pi_0^c(\mathbb{PSC}(M^n))$.

The group occurring in the statement of Theorem 2.1 is essentially Hajduk's obstruction group for the existence of metrics of positive scalar curvature (for more details see §2 and **[H2]**).

Every smooth map $g\colon [0, 1] \to \mathbb{PSC}(M^n)$ induces a concordance between $g(0)$ and $g(1)$ **[GL2**, Proposition 4.43**]**. In other words,