THE FORMAL GROUP OF THE JACOBIAN OF AN ALGEBRAIC CURVE

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In this paper we give an explicit construction of the formal group of the Jacobian of an algebraic curve using a basis for the holomorphic differentials on the curve at a rational non-Weierstrass point. We construct the formal group of the Jacobian of the modular curve $X_0(l)$ and using a result of T. Honda, we prove that this formal group is *p*integral for all but finitely many *p*.

Introduction. Formal group laws have proven to be very useful tools in many areas of mathematics and computer science. In particular, the formal group of an elliptic curve has been used to great effect in elliptic curve theory (for details see for example Silverman [13]) and the use of the formal group of an abelian variety is pervasive in arithmetic and algebraic geometry (see Shatz [11] or Milne [8]). Despite the fact that explicit formulae have been useful in the elliptic curve case, explicit examples of the formal group of other abelian varieties are few.

Recently, Grant [4] and Flynn [3] have independently given explicit constructions of the formal group of the Jacobian of curves of genus two. Grant uses classical formulae for genus two theta functions to give explicit defining equations for the Jacobian and a set of parameters for its group law in a specific \mathbb{P}^8 embedding. His embedding requires that the curve have a Weierstrass point defined over the base field. Flynn's result does not assume the existence of a rational Weierstrass point and thus he must use a \mathbb{P}^{15} embedding of the Jacobian. This paper gives an explicit construction of the formal group of the Jacobian using a basis for the holomorphic differentials on the curve. The construction generalizes that of the formal group of an elliptic curve.

In §I, we review some of the basic facts about higher dimensional formal groups. Hazewinkel [5] is a good reference for the theory of formal groups in general. Section II details the construction of the formal group of the Jacobian. Since this construction depends only on a basis for the holomorphic differentials on the curve at a non-Weierstrass point, it is especially useful in cases where much is known