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THE ENDLICH BAER SPLITTING PROPERTY

THEODORE G. FATICONI

It is well known that projective modules P are characterized by the property that each surjection $M \to P$ of modules splits. For arbitrary modules A one can ask for conditions under which each surjection $A^{(c)} \to A^{(d)}$ will split where c and d are cardinals. Modules with this property are said to have the *Baer splitting property*. If the surjection $A^{(c)} \to A^{(d)}$ splits whenever d is a finite cardinal then A is said to have the *finite Baer splitting property*. If the surjection $A^{(c)} \to A^{(d)}$ splits whenever c and d are finite cardinals then A is said to have the *endlich Baer splitting property*. Albrecht generalizes a theorem of Arnold and Lady by showing that if A satisfies mild hypotheses, then A has the Baer splitting property iff $IA \neq A$ for each proper right ideal $I \subset \mathrm{End}(A)$.

The goal of this paper is to organize what is known about the (finite, endlich) Baer splitting property by generalizing to pairs (A, P) that have the (endlich) Baer splitting property. (See definitions below.) As an application, we show that the torsion-free abelian group of finite rank has the finite Baer splitting property iff it has the endlich Baer splitting property. We cite examples to show that this result is not true of countable modules.

1. Introduction. Throughout this paper, R denotes a fixed but otherwise arbitrary associative ring, A is a right R-module, and $E = \mathrm{End}_R(A)$ denotes the ring of R-module endomorphisms of A. The term *module* will mean right R-module, $\mathscr{M}_R$ denotes the category of modules, and $\mathscr{M}_E$ denotes the category of right E-modules. Let $T_A(\cdot) = \cdot \otimes_E A$ and let $H_A(\cdot) = \mathrm{Hom}_R(A, \cdot)$. The module G is (*finitely*) A*-generated* if there is a (finite) subset $H \subset H_A(G)$ such that $G = \sum\{f(A) | f \in H\}$.

Fix a pair (A, P) of modules, and consider the statements

(I) If $g: G \to P$ is a surjection of modules such that $G' + \ker g = G$ for some A-generated submodule $G' \subset G$, then g is a split surjection.

(I_0) If $g: G \to P$ is a surjection of modules such that $G' + \ker g = G$ for some finitely A-generated submodule $G' \subset G$, then g is a split surjection.

Reinhold Baer has proved that if A is a subgroup of the abelian group $\mathbf{Q}$ of rational numbers then (A, A) satisfies (I), [**15**, Proposition 86.5]. This result, known as Baer's Lemma, has assumed an