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INDEX THEORY AND TOEPLITZ ALGEBRAS ON ONE-PARAMETER SUBGROUPS OF LIE GROUPS

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We form the Toeplitz C^*-algebra $\mathscr{T}(G; X)$ associated to the one-parameter subgroup $\exp(tX)$ defined by a left-invariant vector field X on a compact Lie group G. We compute the K-theory of $\mathscr{T}(G; X)$ and its commutator ideal $\mathscr{C}(G; X)$. We also define an abstract analytical index for $\mathscr{T}(G; X)$ and show that this analytical index can be computed in terms of topological data.

Introduction. Let G be a compact Lie group, let X be a non-zero left-invariant vector field on G, and let $\frac{1}{i}L_X$ denote the Lie derivative with respect to X. The operator $\frac{1}{i}L_X$ extends to an unbounded self-adjoint operator on $L^2(G)$; let P denote the positive spectral projection of $\frac{1}{i}L_X$. Next, each continuous complex-valued function ϕ on G gives rise to a bounded operator M_ϕ on $L^2(G)$ via multiplication. We form a C^*-algebra $\mathscr{T}(G; X)$ that is generated by the set $\{PM_\phi : \phi \in C(G)\}$, and we call this algebra the *Toeplitz algebra* of X. In this paper we study $\mathscr{T}(G; X)$, and in particular, we look at how $\mathscr{T}(G; X)$ depends on the geometry and topology of G and the choice of X.

Our interest in $\mathscr{T}(G; X)$ stems from two sources. First, several authors have recently obtained very nice results relating Toeplitz algebras on flows to the topology of the flows [**2**, **8**]. In our case, the vector field X determines a flow on G, and $\mathscr{T}(G; X)$ is the Toeplitz algebra of this flow.

Another important reason for studying $\mathscr{T}(G; X)$ is that this algebra contains information about the self-adjoint operator $\frac{1}{i}L_X$. In [**4**], the authors used Toeplitz algebra techniques to study self-adjoint operators that are elliptic along the leaves of a foliation. In our case, the cosets of $\exp(tX)$ foliate G, and $\frac{1}{i}L_X$ is elliptic along the leaves of this foliation.

In all of the work mentioned above, the authors required their flows and foliations to be minimal. Our foliations are typically not minimal, and in fact can be very far from being minimal. Thus, we must develop techniques for studying $\mathscr{T}(G; X)$ that do not require minimality.