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BILINEAR OPERATORS ON $L^\infty(G)$ OF LOCALLY COMPACT GROUPS

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Let G and H be compact groups. We study in this paper the space $\mathrm{Bil}^\sigma = \mathrm{Bil}^\sigma(L^\infty(G), L^\infty(H))$. That space consists of all bounded bilinear functionals on $L^\infty(G)\times L^\infty(H)$ that are weak* continuous in each variable separately. We prove, among other things, that Bil^σ is isometrically isomorphic to a closed two-sided ideal in $\mathrm{BM}(G, H)$. In the case of abelian G, H, we show that Bil^σ does not have an approximate identity and that $\widehat{G}\times\widehat{H}$ is dense in the maximal ideal space of Bil^σ. Related results are given.

0. Introduction. Let V and W be Banach spaces over the complex numbers, and let $\mathrm{Bil}(V, W)$ denote the space of bounded bilinear functions $F: V\times W \to C$. Then this is a Banach space under the usual vector space operators and the norm

$$\|F\| = \sup\{|F(x, y)| : x\in V,\ y\in W,\ \|x\| = \|y\| = 1\}.$$

Furthermore $\mathrm{Bil}(V, W)$ may be identified with the dual space of $V\hat{\otimes}W$, the projective tensor product of V and W. When X and Y are locally compact Hausdorff spaces, then elements in $\mathrm{Bil}(C_0(X), C_0(Y))$, also denoted by $\mathrm{BM}(X, Y)$, are called *bimeasures* (see Graham and Schreiber [7] and Gilbert, Ito and Schreiber [**4**]).

If V and W are dual Banach spaces, we let $\mathrm{Bil}^\sigma(V, W)$ denote all $F\in\mathrm{Bil}(V, W)$ such that $x\mapsto F(x, y)$ and $y\mapsto F(x, y)$ are continuous when V and W have the weak*-topology. Then, as readily checked, $\mathrm{Bil}^\sigma(V, W)$ is a norm-closed subspace of $\mathrm{Bil}(V, W)$. It is the purpose of this paper to study $\mathrm{Bil}^\sigma(L^\infty(G), L^\infty(H))$ when G and H are compact groups.

In §1, we shall give some general results on

$$\mathrm{Bil}^\sigma(L^\infty(X, \mu), L^\infty(Y, \nu))$$

when X, Y are locally compact Hausdorff spaces and μ, ν are positive regular Borel measures on X and Y, respectively. In §2, we show that if G and H are compact groups, then $\mathrm{Bil}^\sigma = \mathrm{Bil}^\sigma(L^\infty(G), L^\infty(H))$