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OPTIMAL APPROXIMATION CLASS FOR MULTIVARIATE BERNSTEIN OPERATORS

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For the Bernstein polynomial approximation process on a simplex or a cube, the class of functions yielding optimal approximation will be given. That is, we will find the class of functions for which $\|B_n f - f\|_{C(S)} = O(n^{-1})$ in terms of the behaviour of a certain K-functional. Moreover, this is done in the context of direct and converse results which yields an improvement on such results as well.

1. Introduction. For the simplex S in R^d,

$$S \equiv \left\{ x = (x_1, \dots, x_d) : x_i \geq 0, \ |x| \equiv \sum_{i=1}^{d} x_i \leq 1 \right\}, \tag{1.1}$$

the Bernstein polynomial approximation is given by

$$B_n f = B_n(f, x) \equiv \sum_{\mu/n \in S} P_{n,\mu}(x) f\left(\frac{\mu}{n}\right), \qquad x \in S, \tag{1.2}$$

where $\mu = (m_1, \dots, m_d)$ with m_i integers, and

$$P_{n,\mu}(x) \equiv \frac{n!}{\mu!(n - |\mu|)!} x^{\mu} (1 - |x|)^{n-|\mu|}, \qquad |x| \equiv \sum_{i=1}^{d} x_i, \quad \left(|\mu| \equiv \sum_{i=1}^{d} m_i \right) \tag{1.3}$$

with the convention

$$\mu! = m_1! \cdots m_d! \quad \text{and} \quad x^{\mu} = x_1^{m_1} \cdots x_d^{m_d}.$$

For the cube Q in R^d,

$$Q \equiv \{ x = (x_1, \dots, x_d) : 0 \leq x_i \leq 1 \text{ for } 1 \leq i \leq d \}, \tag{1.4}$$

the Bernstein polynomial approximation is given by

$$\overline{B}_n f = \overline{B}_n(f, x) \equiv \sum_{\mu/n \in Q} \overline{P}_{n,\mu}(x) f\left(\frac{\mu}{n}\right), \qquad x \in Q, \tag{1.5}$$