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SZEGÖ MAPS AND HIGHEST WEIGHT REPRESENTATIONS

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Let G be a connected noncompact simple Lie group with finite center and let K be a maximal compact subgroup of G. Assume the space G/K is Hermitian symmetric. We associate to each irreducible representation τ of K a principal series representation $W(\tau)$ and a G-equivariant Szegö-type integral operator S_τ such that S_τ maps the K-finite vectors in $W(\tau)$ onto an irreducible highest weight $\mathfrak{g}$-module $L(\tau)$. Of primary concern here are those representations τ which are reduction points. For such τ, we construct certain systems $\mathscr{D}_\tau$ of G-equivariant differential operators and then utilize $\mathscr{D}_\tau$ to establish the infinitesimal irreducibility of the image of S_τ.

1. Introduction. Let G be a connected noncompact simple Lie group with finite center and let K be a maximal compact subgroup of G. Assume the space G/K is Hermitian symmetric. The main purpose of this article is to realize each irreducible highest weight representation of G as the image of a G-equivariant quotient map defined on principal series representations. To make this more precise, recall that each irreducible highest weight representation π_τ of G is parametrized by an irreducible unitary representation τ of K. Let $C^\infty(G, \tau)$ denote the space of τ-covariant C^∞-functions on G. We associate to τ a principal series representation $W(\tau)$ and a Szegö map $S_\tau: W(\tau) \to C^\infty(G, \tau)$ having the property that the K-finite vectors in $W(\tau)$ are mapped onto an irreducible $\mathfrak{g}$-module equivalent to the derived action of π_τ. In the case of discrete series and limits thereof, this type of result was proved by Knapp and Wallach **[16]** in the general setting where G is a semisimple equirank Lie group with finite center. The main result here is that the irreducibility of the image of S_τ persists for all highest weight representations. Moreover, for certain τ called reduction points, the irreducibility of Image(S_τ) is proved by showing this space is annihilated by a system $\mathscr{D}_\tau$ of G-equivariant differential operators. The system $\mathscr{D}_\tau$ somewhat parallels the role of the Schmid operator in the Knapp and Wallach result.

The realization of distinguished representations as irreducible images of quotient maps is a recurring theme in the literature which