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DUALITY FOR FINITE BIPARTITE GRAPHS (WITH AN APPLICATION TO II_1 FACTORS)

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Let Γ be a finite graph with bicolored vertices and θ a color-preserving automorphism of Γ. We define the dual graph $\hat{\Gamma} = \hat{\Gamma}(\theta)$ of Γ by θ and the dual $\hat{\theta}$ of θ which is an automorphism of $\hat{\Gamma}$. Under some conditions, $\hat{\hat{\Gamma}}$ is isomorphic to Γ. A bicolored graph gives two weighted graphs. The following pair of graphs treated in Index theory are dual pairs: {Coxeter graph of type A_{2n-3}, D_n}, {$A^{(1)}_{2n-5}$, $D^{(1)}_n$}, {$E^{(1)}_6$, $E^{(1)}_7$}, {$D^{(1)}_l$, $D^{(1)}_{2l-2}$}, and {4-star $S(1, 1, k+1, k+1)$, Γ_k}. The graph of type D_4 or E_6 is self dual, but as a weighted graph, the dual of it is another one.

As applications, we have two kinds of outer automorphisms with the period 2 on inclusions of hyperfinite II_1 factors, one of which gives the inclusion of the crossed products isomorphic to the original one and the other gives the inclusion not isomorphic to the original one.

1. Introduction. Since Jones [7] introduced the index theory for subfactors of finite factors, it is pointed out by several authors that the Dynkin diagrams of Type A, D, E have an important role as an invariant for the classification of the subfactors of the approximately finite dimensional continuous finite factor with index less than 4 (cf. [5], [3], [9] [10], [11], [12], [14]). The relation appears as the diagram for the pair of relative commutant algebras which was defined by Bratteli [2]. He used a finite bipartite graph in order to describe a pair of finite dimensional C^*-algebras (that is, multi-matrix algebras). Today, such a graph is called a Bratteli diagram. On the other hand, in connection with duality theorems in the theory of locally compact groups, many authors following Takesaki [19] obtained duality theorems for the crossed product of operator algebras by locally compact groups.

From such a viewpoint, in this paper, we shall give a duality theorem for certain bipartite finite graphs. We treat a finite graph Γ with bicolored vertices, with an automorphism θ, under which every vertex v has the same color as $\theta(v)$.

Let us consider the graph Γ as a Bratteli diagram of the algebras